J. F. Nash Jr.

This is an unnatural sort of snapshot. It was taken with me in a position such that my head was drawn backwards more than usually is the case.

2/4/48 [illegible]

CARNEGIE INSTITUTE OF TECHNOLOGY — OFFICE OF THE REGISTRAR — PITTSBURGH, PENNSYLVANIA

DAY / EVENING / GRADUATE / SUMMER / REGULAR / SPECIAL	Date of Birth	College	First Name	Middle Name	Last Name
	June 13, 1928	Engineering	John	Forbes	Nash, Jr.
	Date of Admission	Department	Name of Parent or Guardian	Address	
	June 1, 1945	Mathematics	Mr. J. F. Nash	1405 Whitethorn St. Bluefield, W. Va.	
			Name of Preparatory School	Address	
			Beaver High School	Bluefield, W. Va.	

ENTRANCE RECORD

SUBJECTS	Initial Credit	Entrance Condition	Final Credit	SUBJECTS	Initial Credit	Entrance Condition	Final Credit	SUBJECTS	Initial Credit	Entrance Condition	Final Credit
English			4	General Science			1	French			
Algebra			1½	Natural Science			1	German			
Plane Geometry			1	History			3	Spanish			
Solid Geometry—Trigonometry			½ + ½	U. S. History and Civics							
Physics			1	Problems of Democracy							
Chemistry			1	Latin			2	Total			16½

COLLEGE RECORD

AWARDED

SUBJECTS	Subject Number 1st Sem.	Subject Number 2nd Sem.	Attend Hours Per Week	Units Per Sem.	6/5/45 - 1/29/46 1st Sem. Gr.	R.Ex.	2nd Sem. Gr.	R.Ex.	1946 Feb-Sept 1st Sem. Gr.	2nd Sem. Gr.	Oct '46 - May '47 1st Sem. Gr.	2nd Sem. Gr.	R.Ex.	1947 - 1948 1st Sem.	2nd Sem.	19__ - 19__ 1st Sem.	2nd Sem.	Total Sem. Hrs.
General Chemistry	E101	E102	6	9	A		A											
Mathematics	C1	C2	4	12	ad	+	ad	+										
English	C105	C106	2	6	ad	+	ad	+										
Social Relations	C701	C706	2	6	A		A											
~~Eng. Drwg. & Desc. Geom.~~	~~E001~~	~~E002~~	~~6~~	~~9~~														
Hygiene	H1		1	2			A											
Physical Education	H10R	H11	2	0	P		P											
R. O. T. C.	R21	R2	3	5	B													
Physics Rec.	E91	E92	3	8	A		A											
Physics Lab	E91	E92	3	4	A		B											Clear
Mathematics	C3	C4	4	12	A		A											
English	C107		2	6	B													
Social Relations	C715E	C716	3	9					B	A								
Physics	E93		6	12					A									
Quant. Anal. Rec	E112		1	3					B									
Quant. Anal. Lab.	E112		6	6					C									
Seminar	E129	E130	1	1					A	C								
Qual Analysis		E105	6	9			A											
Physical Meas Rec		E73	2	5					A									
Physical Meas Lab		E73	3	4					B									
Physical Education	H13	H14	3	0					P	P								
Eng Drwg. + Desc. Geom	E005a		5	5						C								
Mathematics		C21	3	9			A											
Physical Chem	E125		3	9					B									
Algebra	C23	C24	3	9					A	A								
Vector Analysis		C22	3	9					A									
Social Relations	C725X	C728	3	9						A		A						
Physical Chem Rec		F126	3	9						C								
Physical Chem Lab		F126	6	6						B								
German	C251	C262	3	9							A	A						
Adv. Calculus	S255	S256	4	12							A	A						
Proj. Geometry	S257	S258	3	9							B	A						
Tensor Calculus	S281		3	9							A							
Theor. Mechanics	S261	S262	3	9							A	A						
Chem Option	E146		9							C								
Theory of Functions	GS301	302	3	12							A	A						
Complex Variable	GS303	304	3	12							A	A						
Adv. Calculus	GS319		4	12								A	GS319					
Social Relations	C754		3	9														
Modern Algebra	S273		3	9														
Theor. of Relativity	GS365		3	12														
Quant. Mech.	GS469		3	12														
Function Space	GS373		3	12														
Seminar	S297		1	3														
Total Sem. Hours																		
Total Units per Semester					50		59		67	57	72	84		57				
Q. P. Earned per Semester					189		232		231	174	279	336						
Q. P. Per Unit					3.78		3.93		3.45	3.05	3.88	4.00						

The present status of the student is: Senior in good standing

The student is entitled to an honorable dismissal.

Date of issue: February 3, 1948

E. K. Collins
Recorder / Registrar

THIS TRANSCRIPT NOT VALID UNLESS IT BEARS THE ORIGINAL SIGNATURE OF REGISTRAR OR RECORDER.

EXPLANATION OF RATINGS

Previous to Sept. 1, 1933

A—Excellent
B—Good
C—Satisfactory
D—Fair
E—Poor

Grades below passing are
I—(Incomplete), otherwise satisfactory.
F—(Failure), must fulfill any special requirements and must pass in a second examination.
R—(Repeat).

After Sept. 1, 1933

A—Distinguished
B—Superior
C—Average
D—Passing

Grades below passing are
I—(Incomplete), otherwise satisfactory.
F—(Failure), must fulfill any special requirements and must pass in a second examination.
R—(Repeat).

Ad.—Credit granted for work completed at another institution.
Unit of Work: One hour per week per semester of the student's time, whether recitation, laboratory or home preparation.

January 21, 1948

Mr. John F. Nash, Jr.
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania

Dear Mr. Nash:

Referring to your inquiry regarding teaching positions, it is necessary merely to mention this somewhere in your application. We have no application blanks other than those for the Graduate School. Be sure to have your professors send us letters of recommendation about you -- this is the most important step of all.

Sincerely yours,

SL:MMM

S. Lefschetz

PRINCETON UNIVERSITY THE GRADUATE SCHOOL

NASH, JOHN FORBES, JR. Enrolled 9/20/48 Department MATHEMATICS

Date and place of birth June 13, 1928 Bluefield, W.Va. Single x Married

Bachelor and other degrees B.S. Carnegie Institute of Technology, June 1948; M.S. June 1948; Ph.D. Princeton University, 1950

Previous graduate study

Teaching experience

Address: Princeton B Pyne Tower, G.C.; 193 G.C.

Parent or guardian and address Mr.John F. Nash, 1405 Whitethorn St., Bluefield, West Virginia

1948-49 J.S.K. Fellow in Mathematics

1949-50 Atomic Energy Commission Predoctoral Fellow

A.M.; M.F.A. or M.S.E. granted

Address

PH.D.

French satisfactory May 27, 1949 German satisfactory March 4, 1949

General Examination Passed October 11, 1949

Dissertation subject "Non-Cooperative Games".

Dissertation accepted May 22, 1950

Final Examination Passed May 29, 1950 Degree granted June 13, 1950

Address

NASH, JOHN FORBES, JR. — Department MATHEMATICS

FIRST TERM		SECOND TERM	
1948-1949		1948-1949	
537 Differential Equations - - - - - - - - MacColl	P	502 Mathematical Logic - - - - - - - - - - Church	P
501 Mathematical Logic - - - - - - - - - - Church	P	506 Algebra - - - - - - - - - - - - - - - - Artin	P
525 Algebraic Topology - - - - - - - - - - Steenrod	P	526 Algebraic Topology - - - - - - - - - - Steenrod	P
1949-1950		1949-1950	
539 Partial Differential Equations - - - - Schiffer	G	Convex Sets Seminar - - - - - - - - - - - -Bateman	VG
Seminar on Theory of Games - - - - - - - Gale		Differential Geometry Seminar	
The Problem of Three Bodies and Related Questions (Institute for Advanced Study) Siegel		Algebraic Geometry Seminar	
		Research and work on dissertation - - - - - Tucker	
		526 Algebraic Topology - - - - - - - - - - Steenrod	E

Examiners Report — General Examination		Examiners Report — Final Examination	
Excellent		Excellent	

Background of John Nash *50 from his Graduate Alumni File:

B.S. and M.S. in mathematics from Carnegie Institute of Technology, 1948.

At Princeton:
1948-49: J.S.K. Fellow in Mathematics
1949-50: Atomic Energy Commission Predoctoral Fellow

Ph.D, Princeton University, 1950.

Dissertation entitled: Non-Cooperative Games

In addition, there are clippings from *The Daily Princetonian* from 1994 onward regarding his Nobel

This is the information that may be released to the public. Everything else (save photos) is closed.

NON-COOPERATIVE GAMES

John Nash

Annals of Mathematics
Vol. 54, No. 2, September, 1951

Made in United States of America

Annals of Mathematics
Vol. 54, No. 2, September, 1951

NON-COOPERATIVE GAMES

John Nash

(Received October 11, 1950)

Introduction

Von Neumann and Morgenstern have developed a very fruitful theory of two-person zero-sum games in their book *Theory of Games and Economic Behavior*. This book also contains a theory of n-person games of a type which we would call cooperative. This theory is based on an analysis of the interrelationships of the various coalitions which can be formed by the players of the game.

Our theory, in contradistinction, is based on the *absence* of coalitions in that it is assumed that each participant acts independently, without collaboration or communication with any of the others.

The notion of an *equilibrium point* is the basic ingredient in our theory. This notion yields a generalization of the concept of the solution of a two-person zero-sum game. It turns out that the set of equilibrium points of a two-person zero-sum game is simply the set of all pairs of opposing "good strategies."

In the immediately following sections we shall define equilibrium points and prove that a finite non-cooperative game always has at least one equilibrium point. We shall also introduce the notions of solvability and strong solvability of a non-cooperative game and prove a theorem on the geometrical structure of the set of equilibrium points of a solvable game.

As an example of the application of our theory we include a solution of a simplified three person poker game.

Formal Definitions and Terminology

In this section we define the basic concepts of this paper and set up standard terminology and notation. Important definitions will be preceded by a subtitle indicating the concept defined. The non-cooperative idea will be implicit, rather than explicit, below.

Finite Game:

For us an *n-person game* will be a set of n *players*, or *positions*, each with an associated finite set of *pure strategies*; and corresponding to each player, i, a *payoff function*, p_i, which maps the set of all n-tuples of pure strategies into the real numbers. When we use the term *n-tuple* we shall always mean a set of n items, with each item associated with a different player.

Mixed Strategy, s_i :

A *mixed strategy* of player i will be a collection of non-negative numbers which have unit sum and are in one to one correspondence with his pure strategies.

We write $s_i = \sum_\alpha c_{i\alpha}\pi_{i\alpha}$ with $c_{i\alpha} \geqq 0$ and $\sum_\alpha c_{i\alpha} = 1$ to represent such a mixed strategy, where the $\pi_{i\alpha}$'s are the pure strategies of player i. We regard the s_i's as points in a simplex whose vertices are the $\pi_{i\alpha}$'s. This simplex may be regarded as a convex subset of a real vector space, giving us a natural process of linear combination for the mixed strategies.

We shall use the suffixes i, j, k for players and α, β, γ to indicate various pure strategies of a player. The symbols s_i, t_i, and r_i, etc. will indicate mixed strategies; $\pi_{i\alpha}$ will indicate the i^{th} player's α^{th} pure strategy, etc.

Payoff function, p_i:

The payoff function, p_i, used in the definition of a finite game above, has a unique extension to the n-tuples of mixed strategies which is linear in the mixed strategy of each player [n-linear]. This extension we shall also denote by p_i, writing $p_i(s_1, s_2, \cdots, s_n)$.

We shall write $\mathfrak{s}$ or $\mathfrak{t}$ to denote an n-tuple of mixed strategies and if $\mathfrak{s} = (s_1, s_2, \cdots, s_n)$ then $p_i(\mathfrak{s})$ shall mean $p_i(s_1, s_2, \cdots, s_n)$. Such an n-tuple, $\mathfrak{s}$, will also be regarded as a point in a vector space, the product space of the vector spaces containing the mixed strategies. And the set of all such n-tuples forms, of course, a convex polytope, the product of the simplices representing the mixed strategies.

For convenience we introduce the substitution notation $(\mathfrak{s}; t_i)$ to stand for $(s_1, s_2, \cdots, s_{i-1}, t_i, s_{i+1}, \cdots, s_n)$ where $\mathfrak{s} = (s_1, s_2, \cdots, s_n)$. The effect of successive substitutions $((\mathfrak{s}; t_i); r_j)$ we indicate by $(\mathfrak{s}; t_i; r_j)$, etc.

Equilibrium Point:

An n-tuple $\mathfrak{s}$ is an *equilibrium point* if and only if for every i

(1) $$p_i(\mathfrak{s}) = \max_{\text{all } r_i\text{'s}} [p_i(\mathfrak{s}; r_i)].$$

Thus an equilibrium point is an n-tuple $\mathfrak{s}$ such that each player's mixed strategy maximizes his payoff if the strategies of the others are held fixed. Thus each player's strategy is optimal against those of the others. We shall occasionally abbreviate equilibrium point by eq. pt.

We say that a mixed strategy s_i *uses* a pure strategy $\pi_{i\alpha}$ if $s_i = \sum_\beta c_{i\beta}\pi_{i\beta}$ and $c_{i\alpha} > 0$. If $\mathfrak{s} = (s_1, s_2, \cdots, s_n)$ and s_i uses $\pi_{i\alpha}$ we also say that $\mathfrak{s}$ uses $\pi_{i\alpha}$.

From the linearity of $p_i(s_1, \cdots, s_n)$ in s_i,

(2) $$\max_{\text{all } r_i\text{'s}} [p_i(\mathfrak{s}; r_i)] = \max_\alpha [p_i(\mathfrak{s}; \pi_{i\alpha})].$$

We define $p_{i\alpha}(\mathfrak{s}) = p_i(\mathfrak{s}; \pi_{i\alpha})$. Then we obtain the following trivial necessary and sufficient condition for $\mathfrak{s}$ to be an equilibrium point:

(3) $$p_i(\mathfrak{s}) = \max_\alpha p_{i\alpha}(\mathfrak{s}).$$

If $\mathfrak{s} = (s_1, s_2, \cdots, s_n)$ and $s_i = \sum_\alpha c_{i\alpha}\pi_{i\alpha}$ then $p_i(\mathfrak{s}) = \sum_\alpha c_{i\alpha}p_{i\alpha}(\mathfrak{s})$, consequently for (3) to hold we must have $c_{i\alpha} = 0$ whenever $p_{i\alpha}(\mathfrak{s}) < \max_\beta p_{i\beta}(\mathfrak{s})$, which is to say that $\mathfrak{s}$ does not use $\pi_{i\alpha}$ unless it is an optimal pure strategy for player i. So we write

(4) $$\text{if } \pi_{i\alpha} \text{ is used in } \mathfrak{s} \text{ then } p_{i\alpha}(\mathfrak{s}) = \max_\beta p_{i\beta}(\mathfrak{s})$$

as another necessary and sufficient condition for an equilibrium point.

Since a criterion (3) for an eq. pt. can be expressed by the equating of n pairs of continuous functions on the space of n-tuples $\mathfrak{s}$ the eq. pts. obviously form a closed subset of this space. Actually, this subset is formed from a number of pieces of algebraic varieties, cut out by other algebraic varieties.

Existence of Equilibrium Points

A proof of this existence theorem based on Kakutani's generalized fixed point theorem was published in Proc. Nat. Acad. Sci. U. S. A., 36, pp. 48–49. The proof given here is a considerable improvement over that earlier version and is based directly on the Brouwer theorem. We proceed by constructing a continuous transformation T of the space of n-tuples such that the fixed points of T are the equilibrium points of the game.

THEOREM 1. *Every finite game has an equilibrium point.*

PROOF. Let $\mathfrak{s}$ be an n-tuple of mixed strategies, $p_i(\mathfrak{s})$ the corresponding pay-off to player i, and $p_{i\alpha}(\mathfrak{s})$ the pay-off to player i if he changes to his α^{th} pure strategy $\pi_{i\alpha}$ and the others continue to use their respective mixed strategies from $\mathfrak{s}$. We now define a set of continuous functions of $\mathfrak{s}$ by

$$\varphi_{i\alpha}(\mathfrak{s}) = \max (0, p_{i\alpha}(\mathfrak{s}) - p_i(\mathfrak{s}))$$

and for each component s_i of $\mathfrak{s}$ we define a modification s_i' by

$$s_i' = \frac{s_i + \sum_\alpha \varphi_{i\alpha}(\mathfrak{s})\pi_{i\alpha}}{1 + \sum_\alpha \varphi_{i\alpha}(\mathfrak{s})},$$

calling $\mathfrak{s}'$ the n-tuple $(s_1', s_2', s_3' \cdots s_n')$.

We must now show that the fixed points of the mapping $T: \mathfrak{s} \to \mathfrak{s}'$ are the equilibrium points.

First consider any n-tuple $\mathfrak{s}$. In $\mathfrak{s}$ the i^{th} player's mixed strategy s_i will use certain of his pure strategies. Some one of these strategies, say $\pi_{i\alpha}$, must be "least profitable" so that $p_{i\alpha}(\mathfrak{s}) \leqq p_i(\mathfrak{s})$. This will make $\varphi_{i\alpha}(\mathfrak{s}) = 0$.

Now if this n-tuple $\mathfrak{s}$ happens to be fixed under T the proportion of $\pi_{i\alpha}$ used in s_i must not be decreased by T. Hence, for all β's, $\varphi_{i\beta}(\mathfrak{s})$ must be zero to prevent the denominator of the expression defining s_i' from exceeding 1.

Thus, if $\mathfrak{s}$ is fixed under T, for any i and β $\varphi_{i\beta}(\mathfrak{s}) = 0$. This means no player can improve his pay-off by moving to a pure strategy $\pi_{i\beta}$. But this is just a criterion for an eq. pt. [see (2)].

Conversely, if $\mathfrak{s}$ is an eq. pt. it is immediate that all φ's vanish, making $\mathfrak{s}$ a fixed point under T.

Since the space of n-tuples is a cell the Brouwer fixed point theorem requires that T must have at least one fixed point $\mathfrak{s}$, which must be an equilibrium point.

Symmetries of Games

An *automorphism*, or *symmetry*, of a game will be a permutation of its pure strategies which satisfies certain conditions, given below.

If two strategies belong to a single player they must go into two strategies belonging to a single player. Thus if ϕ is the permutation of the pure strategies it induces a permutation ψ of the players.

Each n-tuple of pure strategies is therefore permuted into another n-tuple of pure strategies. We may call χ the induced permutation of these n-tuples. Let ξ denote an n-tuple of pure strategies and $p_i(\xi)$ the payoff to player i when the n-tuple ξ is employed. We require that if

$$j = i^{\psi} \qquad \text{then } p_j(\xi^{\chi}) = p_i(\xi)$$

which completes the definition of a symmetry.

The permutation ϕ has a unique linear extension to the mixed strategies. If

$$s_i = \sum_\alpha c_{i\alpha}\pi_{i\alpha} \quad \text{we define} \quad (s_i)^{\phi} = \sum_\alpha c_{i\alpha}(\pi_{i\alpha})^{\phi}.$$

The extension of ϕ to the mixed strategies clearly generates an extension of χ to the n-tuples of mixed strategies. We shall also denote this by χ.

We define a *symmetric n-tuple* $\mathfrak{s}$ of a game by $\mathfrak{s}^{\chi} = \mathfrak{s}$ for all χ's

THEOREM 2. *Any finite game has a symmetric equilibrium point.*

PROOF. First we note that $s_{i0} = \sum_\alpha \pi_{i\alpha}/\sum_\alpha 1$ has the property $(s_{i0})^{\phi} = s_{j0}$ where $j = i^{\psi}$, so that the n-tuple $\mathfrak{s}_0 = (s_{10}, s_{20}, \cdots, s_{n0})$ is fixed under any χ; hence any game has at least one symmetric n-tuple.

If $\mathfrak{s} = (s_1, \cdots, s_n)$ and $\mathfrak{t} = (t_1, \cdots, t_n)$ are symmetric then

$$\frac{\mathfrak{s} + \mathfrak{t}}{2} = \left(\frac{s_1 + t_1}{2}, \frac{s_2 + t_2}{2}, \cdots, \frac{s_n + t_n}{2}\right)$$

is also symmetric because $\mathfrak{s}^{\chi} = \mathfrak{s} \leftrightarrow s_j = (s_i)^{\phi}$, where $j = i^{\psi}$, hence

$$\frac{s_j + t_j}{2} = \frac{(s_i)^{\phi} + (t_i)^{\phi}}{2} = \left(\frac{s_i + t_i}{2}\right)^{\phi},$$

hence

$$\left(\frac{\mathfrak{s} + \mathfrak{t}}{2}\right)^{\chi} = \frac{\mathfrak{s} + \mathfrak{t}}{2}.$$

This shows that the set of symmetric n-tuples is a convex subset of the space of n-tubles since it is obviously closed.

Now observe that the mapping $T: \mathfrak{s} \to \mathfrak{s}'$ used in the proof of the existence theorem was intrinsically defined. Therefore, if $\mathfrak{s}_2 = T\mathfrak{s}_1$ and χ is derived from an automorphism of the game we will have $\mathfrak{s}_2^{\chi} = T\mathfrak{s}_1^{\chi}$. If $\mathfrak{s}_1$ is symmetric $\mathfrak{s}_1^{\chi} = \mathfrak{s}_1$ and therefore $\mathfrak{s}_2^{\chi} = T\mathfrak{s}_1 = \mathfrak{s}_2$. Consequently this mapping maps the set of symmetric n-tuples into itself.

Since this set is a cell there must be a symmetric fixed point $\mathfrak{s}$ which must be a symmetric equilibrium point.

Solutions

We define here solutions, strong solutions, and sub-solutions. A non-cooperative game does not always have a solution, but when it does the solution is unique. Strong solutions are solutions with special properties. Sub-solutions always exist and have many of the properties of solutions, but lack uniqueness.

S_1 will denote a set of mixed strategies of player i and $\mathfrak{S}$ a set of n-tuples of mixed strategies.

Solvability:

A game is *solvable* if its set, $\mathfrak{S}$, of equilibrium points satisfies the condition

$$(\mathbf{t};\ r_i) \in \mathfrak{S} \quad \text{and} \quad \mathfrak{s} \in \mathfrak{S} \rightarrow (\mathfrak{s};\ r_i) \in \mathfrak{S} \qquad \text{for all } i\text{'s}. \tag{5}$$

This is called the *interchangeability* condition. The *solution* of a solvable game is its set, $\mathfrak{S}$, of equilibrium points.

Strong Solvability:

A game is *strongly solvable* if it has a solution, $\mathfrak{S}$, such that for all i's

$$\mathfrak{s} \in \mathfrak{S} \quad \text{and} \quad p_i(\mathfrak{s};\ r_i) = p_i(\mathfrak{s}) \rightarrow (\mathfrak{s};\ r_i) \in \mathfrak{S}$$

and then $\mathfrak{S}$ is called a *strong solution*.

Equilibrium Strategies:

In a solvable game let S_i be the set of all mixed strategies s_i such that for some $\mathbf{t}$ the n-tuple $(\mathbf{t};\ s_i)$ is an equilibrium point. [s_i is the i^{th} component of some equilibrium point.] We call S_i the set of *equilibrium strategies* of player i.

Sub-solutions:

If $\mathfrak{S}$ is a subset of the set of equilibrium points of a game and satisfies condition (1); and if $\mathfrak{S}$ is maximal relative to this property then we call $\mathfrak{S}$ a *sub-solution*.

For any sub-solution $\mathfrak{S}$ we define the i^{th} *factor set*, S_i, as the set of all s_i's such that $\mathfrak{S}$ contains $(\mathbf{t};\ s_i)$ for some $\mathbf{t}$.

Note that a sub-solution, when unique, is a solution; and its factor sets are the sets of equilibrium strategies.

THEOREM 3. *A sub-solution, $\mathfrak{S}$, is the set of all n-tuples $(s_1, s_2, \cdots, s_n)$ such that each $s_i \in S_i$ where S_i is the i^{th} factor set of $\mathfrak{S}$. Geometrically, $\mathfrak{S}$ is the product of its factor sets.*

PROOF. Consider such an n-tuple $(s_1, s_2, \cdots, s_n)$. By definition $\exists\ \mathbf{t}_1, \mathbf{t}_2, \cdots, \mathbf{t}_n$ such that for each i $(\mathbf{t}_i;\ s_i) \in \mathfrak{S}$. Using the condition (5) $n-1$ times we obtain successively $(\mathbf{t}_1;\ s_1) \in \mathfrak{S}$, $(\mathbf{t}_1;\ s_1;\ s_2) \in \mathfrak{S}$, $\cdots$, $(\mathbf{t}_1;\ s_1;\ s_2;\ \cdots;\ s_n) \in \mathfrak{S}$ and the last is simply $(s_1, s_2, \cdots, s_n) \in \mathfrak{S}$, which we needed to show.

THEOREM 4. *The factor sets $S_1, S_2, \cdots, S_n$ of a sub-solution are closed and convex as subsets of the mixed strategy spaces.*

PROOF. It suffices to show two things:

(a) if s_i and $s_i' \in S_i$ then $s_i^* = (s_i + s_i')/2 \in S_i$; (b) if s_i^* is a limit point of S_i then $s_i^* \in S_i$.

Let $\mathbf{t} \in \mathfrak{S}$. Then we have $p_j(\mathbf{t};\ s_i) \geqq p_j(\mathbf{t};\ s_i;\ r_j)$ and $p_j(\mathbf{t};\ s_i') \geqq p_j(\mathbf{t};\ s_i';\ r_j)$ for any r_j, by using the criterion of (1) for an eq. pt. Adding these inequalities, using the linearity of $p_j(s_1, \cdots, s_n)$ in s_i, and dividing by 2, we get $p_j(\mathbf{t};\ s_i^*) \geqq$

$p_j(\mathbf{t}; s_i^* ; r_j)$ since $s_i^* = (s_i + s_i')/2$. From this we know that $(\mathbf{t}; s_i)$ is an eq. pt. for any $\mathbf{t} \in \mathfrak{S}$. If the set of all such eq. pts. $(\mathbf{t}; s_i^*)$ is added to $\mathfrak{S}$ the augmented set clearly satisfies condition (5), and since $\mathfrak{S}$ was to be maximal it follows that $s_i^* \in S_i$.

To attack (b) note that the n-tuple $(\mathbf{t}; s_i^{\#})$, where $\mathbf{t} \in \mathfrak{S}$, will be a limit point of the set of n-tuples of the form $(\mathbf{t}; s_i)$ where $s_i \in S_i$, since $s_i^{\#}$ is a limit point of S_i. But this set is a set of eq. pts. and hence any point in its closure is an eq. pt., since the set of all eq. pts. is closed. Therefore $(\mathbf{t}; s_i^{\#})$ is an eq. pt. and hence $s_i^{\#} \in S_i$ from the same argument as for s_i^*.

Values:

Let $\mathfrak{S}$ be the set of equilibrium points of a game. We define

$$v_i^+ = \max_{\mathfrak{s} \in \mathfrak{S}} [p_i(\mathfrak{s})], \qquad v_i^- = \min_{\mathfrak{s} \in \mathfrak{S}} [p_i(\mathfrak{s})].$$

If $v_i^+ = v_i^-$ we write $v_i = v_i^+ = v_i^-$ · v_i^+ is the *upper value* to player i of the game; v_i^- the *lower value*; and v_i the *value*, if it exists.

Values will obviously have to exist if there is but one equilibrium point.

One can define *associated values* for a sub-solution by restricting $\mathfrak{S}$ to the eq. pts. in the sub-solution and then using the same defining equations as above.

A two-person zero-sum game is always solvable in the sense defined above. The sets of equilibrium strategies S_1 and S_2 are simply the sets of "good" strategies. Such a game is not generally strongly solvable; strong solutions exist only when there is a "saddle point" in *pure* strategies.

Simple Examples

These are intended to illustrate the concepts defined in the paper and display special phenomena which occur in these games.

The first player has the roman letter strategies and the payoff to the left, etc.

Ex. 1	5	$a\alpha$	−3	Solution $\left(\frac{9}{16}a + \frac{7}{16}b, \frac{7}{17}\alpha + \frac{10}{17}\beta\right)$
	−4	$a\beta$	4	
	−5	$b\alpha$	5	$v_1 = \frac{-5}{17}, v_2 = +\frac{1}{2}$
	3	$b\beta$	−4	
Ex. 2	1	$a\alpha$	1	Strong Solution (b, β)
	−10	$a\beta$	10	
	10	$b\alpha$	−10	$v_1 = v_2 = -1$
	−1	$b\beta$	−1	
Ex. 3	1	$a\alpha$	1	Unsolvable; equilibrium points (a, α), (b, β),
	−10	$a\beta$	−10	and $\left(\frac{a}{2} + \frac{b}{2}, \frac{\alpha}{2} + \frac{\beta}{2}\right)$. The strategies in the last case
	−10	$b\alpha$	−10	
	1	$b\beta$	1	have maxi-min and mini-max properties.
Ex. 4	1	$a\alpha$	1	Strong Solution: all pairs of mixed strategies.
	0	$a\beta$	1	
	1	$b\alpha$	0	$v_1^+ = v_2^+ = 1, v_1^- = v_2^- = 0.$
	0	$b\beta$	0	

Ex. 5	1	$a\alpha$	2	Unsolvable; eq. pts. (a, α), (b, β) and
	-1	$a\beta$	-4	$\left(\frac{1}{4}a + \frac{3}{4}b, \frac{3}{8}\alpha + \frac{5}{8}\beta\right)$. However, empirical tests
	-4	$b\alpha$	-1	
	2	$b\beta$	1	show a tendency toward (a, α).
Ex. 6	1	$a\alpha$	1	Eq. pts.: (a, α) and (b, β), with (b, β) an example of
	0	$a\beta$	0	instability.
	0	$b\alpha$	0	
	0	$b\beta$	0	

Geometrical Form of Solutions

In the two-person zero-sum case it has been shown that the set of "good" strategies of a player is a convex polyhedral subset of his strategy space. We shall obtain the same result for a player's set of equilibrium strategies in any solvable game.

THEOREM 5. *The sets $S_1, S_2, \cdots, S_n$ of equilibrium strategies in a solvable game are polyhedral convex subsets of the respective mixed strategy spaces.*

PROOF. An n-tuple $\mathfrak{s}$ will be an equilibrium point if and only if for every i

$$p_i(\mathfrak{s}) = \max_{\alpha} p_{i\alpha}(\mathfrak{s}) \tag{6}$$

which is condition (3). An equivalent condition is for every i and α

$$p_i(\mathfrak{s}) - p_{i\alpha}(\mathfrak{s}) \geqq 0. \tag{7}$$

Let us now consider the form of the set S_j of equilibrium strategies, s_j, of player j. Let $\mathbf{t}$ be any equilibrium point, then $(\mathbf{t}; s_j)$ will be an equilibrium point if and only if $s_j \in S_j$, from Theorem 2. We now apply conditions (2) to $(\mathbf{t}; s_j)$, obtaining

$$s_j \in S_j \longleftrightarrow \text{for all } i, \alpha \qquad p_i(\mathbf{t}; s_j) - p_i\alpha(\mathbf{t}; s_j) \geqq 0. \tag{8}$$

Since p_i is n-linear and $\mathbf{t}$ is constant these are a set of linear inequalities of the form $F_{i\alpha}(s_j) \geqq 0$. Each such inequality is either satisfied for all s_j or for those lying on and to one side of some hyperplane passing through the strategy simplex. Therefore, the complete set [which is finite] of conditions will all be satisfied simultaneously on some convex polyhedral subset of player j's strategy simplex. [Intersection of half-spaces.]

As a corollary we may conclude that S_j is the convex closure of a finite set of mixed strategies [vertices].

Dominance and Contradiction Methods

We say that s_i' dominates s_i if $p_i(\mathbf{t}; s_i') > p_i(\mathbf{t}; s_i)$ for every $\mathbf{t}$.

This amounts to saying that s_i' gives player i a higher payoff than s_i no matter what the strategies of the other players are. To see whether a strategy s_i' dominates s_i it suffices to consider only pure strategies for the other players because of the n-linearity of p_i.

It is obvious from the definitions that *no equilibrium point can involve a dominated strategy s_i.*

The domination of one mixed strategy by another will always entail other dominations. For suppose s_i' dominates s_i and t_i uses all of the pure strategies which have a higher coefficient in s_i than in s_i'. Then for a small enough ρ

$$t_i' = t_i + \rho(s_i' - s_i)$$

is a mixed strategy; and t_i dominates t_i' by linearity.

One can prove a few properties of the set of undominated strategies. It is simply connected and is formed by the union of some collection of faces of the strategy simplex.

The information obtained by discovering dominances for one player may be of relevance to the others, insofar as the elimination of classes of mixed strategies as possible components of an equilibrium point is concerned. For the t's whose components are all undominated are all that need be considered and thus eliminating some of the strategies of one player may make possible the elimination of a new class of strategies for another player.

Another procedure which may be used in locating equilibrium points is the contradiction-type analysis. Here one assumes that an equilibrium point exists having component strategies lying within certain regions of the strategy spaces and proceeds to deduce further conditions which must be satisfied if the hypothesis is true. This sort of reasoning may be carried through several stages to eventually obtain a contradiction indicating that there is no equilibrium point satisfying the initial hypothesis.

A Three-Man Poker Game

As an example of the application of our theory to a more or less realistic case we include the simplified poker game given below. The rules are as follows:

(a) The deck is large, with equally many *high* and *low* cards, and a hand consists of one card.

(b) Two chips are used to ante, open, or call.

(c) The players play in rotation and the game ends after all have passed or after one player has opened and the others have had a chance to call.

(d) If no one bets the antes are retrieved.

(e) Otherwise the pot is divided equally among the highest hands which have bet.

We find it more satisfactory to treat the game in terms of quantities we call "behavior parameters" than in the normal form of *Theory of Games and Economic Behavior*. In the normal form representation two mixed strategies of a player may be equivalent in the sense that each makes the individual choose each available course of action in each particular situation requiring action on his part with the same frequency. That is, they represent the same behavior pattern on the part of the individual.

Behavior parameters give the probabilities of taking each of the various possible actions in each of the various possible situations which may arise. Thus they describe behavior patterns.

In terms of behavior parameters the strategies of the players may be represented as follows, assuming that since there is no point in passing with a *high* card at one's last opportunity to bet that this will not be done. The greek letters are the probabilities of the various acts.

	First Moves	Second Moves
I	α Open on *high* β Open on *low*	κ Call III on *low* λ Call II on *low* μ Call II and III on *low*
II	γ Call I on *low* δ Open on *high* ε Open on *low*	ν Call III on *low* ξ Call III and I on *low*
III	ζ Call I and II on *low* η Open on *low* θ Call I on *low* ι Call II on *low*	Player III never gets a second move

We locate all possible equilibrium points by first showing that most of the greek parameters must vanish. By dominance mainly with a little contradiction-type analysis β is eliminated and with it go γ, ζ, and θ by dominance. Then contradictions eliminate μ, ξ, ι, λ, κ, and ν in that order. This leaves us with α, δ, ε, and η. Contradiction analysis shows that none of these can be zero or one and thus we obtain a system of simultaneous algebraic equations. The equations happen to have but one solution with the variables in the range (0, 1). We get

$$\alpha = \frac{21 - \sqrt{321}}{10}, \qquad \eta = \frac{5\alpha + 1}{4}, \qquad \delta = \frac{5 - 2\alpha}{5 + \alpha}, \qquad \varepsilon = \frac{4\alpha - 1}{\alpha + 5}.$$

These yield $\alpha = .308$, $\eta = .635$, $\delta = .826$, and $\varepsilon = .044$. Since there is only one equilibrium point the game has values; these are

$$v_1 = -.147 = -\frac{(1 + 17\alpha)}{8(5 + \alpha)}, \qquad v_2 = -.096 = -\frac{1 - 2\alpha}{4},$$

and

$$v_3 = .243 = \frac{79}{40}\left(\frac{1 - \alpha}{5 + \alpha}\right).$$

A more complete investigation of this poker game is published in Annals of Mathematics Study No. 24, *Contributions to the Theory of Games*. There the solution is studied as the ratio of ante to bet varies, and the potentialities of coalitions are investigated.

Applications

The study of n-person games for which the accepted ethics of fair play imply non-cooperative playing is, of course, an obvious direction in which to apply this theory. And poker is the most obvious target. The analysis of a more realistic poker game than our very simple model should be quite an interesting affair.

The complexity of the mathematical work needed for a complete investigation increases rather rapidly, however, with increasing complexity of the game; so that analysis of a game much more complex than the example given here might only be feasible using approximate computational methods.

A less obvious type of application is to the study of cooperative games. By a cooperative game we mean a situation involving a set of players, pure strategies, and payoffs as usual; but with the assumption that the players can and will collaborate as they do in the von Neumann and Morgenstern theory. This means the players may communicate and form coalitions which will be enforced by an umpire. It is unnecessarily restrictive, however, to assume any transferability or even comparability of the payoffs [which should be in utility units] to different players. Any desired transferability can be put into the game itself instead of assuming it possible in the extra-game collaboration.

The writer has developed a "dynamical" approach to the study of cooperative games based upon reduction to non-cooperative form. One proceeds by constructing a model of the pre-play negotiation so that the steps of negotiation become moves in a larger non-cooperative game [which will have an infinity of pure strategies] describing the total situation.

This larger game is then treated in terms of the theory of this paper [extended to infinite games] and if values are obtained they are taken as the values of the cooperative game. Thus the problem of analyzing a cooperative game becomes the problem of obtaining a suitable, and convincing, non-cooperative model for the negotiation.

The writer has, by such a treatment, obtained values for all finite two person cooperative games, and some special n-person games.

Acknowledgements

Drs. Tucker, Gale, and Kuhn gave valuable criticism and suggestions for improving the exposition of the material in this paper. David Gale suggested the investigation of symmetric games. The solution of the Poker model was a joint project undertaken by Lloyd S. Shapley and the author. Finally, the author was sustained financially by the Atomic Energy Commission in the period 1949–50 during which this work was done.

Bibliography

(1) von Neumann, Morgenstern, Theory of Games and Economic Behavior, Princeton University Press, 1944.

(2) J. F. Nash, Jr., *Equilibrium Points in N-Person Games*, Proc. Nat. Acad. Sci. U. S. A. 36 (1950) 48–49.

(3) J. F. Nash, L. S. Shapley, A Simple Three-Person Poker Game, Annals of Mathematics Study No. 24, Princeton University Press, 1950.

(4) John Nash, *Two Person Cooperative Games*, to appear in Econometrica.

(5) H. W. Kuhn, *Extensive Games*, Proc. Nat. Acad. Sci. U. S. A., 36 (1950) 570–576.

I accept readmission to the graduate school.

John F. Nash, Jr.

May 2, 1949

Form letter 4-19-49

APR 13 1949

PRINCETON UNIVERSITY
THE GRADUATE SCHOOL

Readmitted
Apr 15/49

Date April 11, 1949

NS7

I hereby make application for re-admission to the Graduate School of Princeton University in the

Department of Mathematics

for the academic year 1949 - 1950

Full Name John Forbes Nash, Jr.

Address G.C., Princeton

NOTE—Applications for re-admission must be filed not later than April 20th of each year.

PRINCETON, N.J.
APR 12
5-PM
1949

THIS SIDE OF CARD IS FOR ADDRESS

THE DEAN OF THE GRADUATE SCHOOL

PRINCETON, NEW JERSEY

CARNEGIE INSTITUTE OF TECHNOLOGY
SCHENLEY PARK
PITTSBURGH 13, PENNSYLVANIA

DEPARTMENT OF MATHEMATICS
COLLEGE OF ENGINEERING AND SCIENCE

February 11, 1948

Professor S. Lefschetz
Department of Mathematics
Princeton University
Princeton, N. J.

Dear Professor Lefschetz:

This is to recommend Mr. John F. Nash, Jr. who has applied for entrance to the graduate college at Princeton.

Mr. Nash is nineteen years old and is graduating from Carnegie Tech in June. He is a mathematical genius.

Yours sincerely,

Richard J Duffin

Richard J. Duffin

RJD:hl

2/2/48 ack.
AMD

FEB 2 1948

x 50

Form letter 4-1-48 about languages, etc

R

PRINCETON UNIVERSITY
THE GRADUATE SCHOOL

Application for Admission

Date ......January 24, 1948......

Department of Proposed Study* ......Mathematics......

Name of Applicant in Full (block letters):

......JOHN FORBES NASH, JR......

Present Post Office Address:

Carnegie Institute of Technology, Pittsburgh, Pa.

Permanent or Home Address:

1405 Whitethorn St., Bluefield W. Va.

Date and place of birth: ......June 13, 1928 ; Above location......

Name and address of parent or nearest relative:

......John Forbes Nash......

......at my HOME ADDRESS......

Condition of general health ......good......

of eyesight fair [I do not use glasses], of hearing ......good......

A medical certificate on the attached blank is required of each applicant, to be forwarded by the doctor to the Department of Health, Princeton University.

Colleges or universities attended, with period of attendance at each, and degrees received, with dates: (For example: Haverford College 1927-1931, A.B. 1931; University of Chicago 1931-1932, A.M. 1932).

Bluefield College 1944-45 ; Supplementary to high school

Carnegie Inst. of Tech. 1945-48 ; to get B.S., M.S. in 1948

Service Record ......

Application for Fellowship or Scholarship

All applications for Fellowships and Scholarships must be received in the Office of the Graduate School by March 1. Ordinarily the fellowships and junior fellowships are awarded to men who have had a year or more of graduate study in a graduate school, and the scholarships, carrying free tuition, to men who have not had such study. If a candidate, in answering the first question below, indicates that he wishes to be considered only for a fellowship, it will be the understanding of this office that he does not wish to be considered for a scholarship, if a fellowship is not available.

State whether you are hereby applying for either a Fellowship or a Scholarship: ......both [either]

If you are applying elsewhere also, give the names of the institutions: ......Chicago, Michigan,

Harvard, perhaps M.I.T. ......

This information will not prejudice the consideration of this application at Princeton.

* For graduate work in the Woodrow Wilson School of Public and International Affairs applicants should apply for special admission forms.

EXTRACT FROM UNIVERSITY REGULATIONS

1. The total enrolment of graduate students is limited.

All applications for admission to the Graduate School of Princeton University are received subject to whatever provisions may determine the mode of limiting the enrolment of graduate students. Admissions are limited to male students.

2. *In every case admission is granted for not more than one academic year.* An application for readmission is necessary for every subsequent year. All applications for admission and for readmission are considered on or about April 20 of each year, except that those who are awarded Princeton fellowships or scholarships for an academic year are granted admission for that year at the time of the award. An application received after admissions have been determined will be placed on file for consideration in case of a vacancy under the limitation of enrolment. Applications received after June 15 may not be acted upon before the opening of the University in September.

3. A bachelor's degree in a broad program of general education, granted by a college or university of recognized standing, is normally requisite for admission, but is not sufficient of itself. An examination of an applicant's academic record is made to determine whether he has established a strong affirmative case in regard to the character of his general education and his fitness for graduate work in his proposed subject of study. In admissions regard is given to character and promise as well as to scholastic attainment. In every case admission is determined, under the limitation of enrolment, after a comparison of the relative merits of all the candidates who wish to pursue their studies in the same field or department.

Any one admitted to the Princeton Graduate School should, before beginning his graduate study, acquaint himself with the full text of the Regulations as printed in the opening pages of the General Information Bulletin.

Applicants will be notified of the result of their applications as soon as possible after April 25. To those who are admitted will be sent a form on which application may be made for residence in the buildings of the Graduate College.

INFORMATION TO BE SUPPLIED BY EVERY APPLICANT

Submit with this application 4 personal photographs of the size approximately 2½ x 2¾ and an official transcript of your undergraduate record (including entrance credits and honors conferred), obtained from the proper official of the institution which granted the degree; also an official transcript of graduate work completed, if any. Each transcript should be accompanied by a key to the grading system employed.

If you have taken the Graduate Record Examination, a copy of the results should be sent with your application.

Submit also a detailed statement of the courses which you have taken in the subject in which you are proposing to do graduate work. The statement about each course should contain a list of the subjects treated in the course (about three lines), the books used, and an indication as to how the course was conducted, whether by lecture, recitation, or a combination of these, or laboratory work, and whether there were regular conferences between you and the instructor in charge. (It is not necessary to supply this information concerning courses taken in Princeton University.)

Give the names (with titles) and addresses of several persons whom you are asking to write intimately and confidentially about your qualifications. The list should include two of your college or university teachers, and the persons most competent to speak of your work in any positions which you have held. Such letters are not necessary in the case of a graduate of Princeton, unless he has carried on studies elsewhere. These personal letters should be sent by the writers directly to the Dean of the Graduate School, and reach him by March 1, if you are applying for a fellowship or graduate scholarship.

√ Prof. J.L. Synge, B.A., M.A., Sc.D., Dept. Head

√ Prof. J.B. Rosenbach, A.B., M.S., Assist. Dept. Head

√ Assoc. Prof. D. Moskovitz, B.S., M.S., Ph.D.

All of Carnegie Mathematics Dept.

PRINCETON UNIVERSITY THE GRADUATE SCHOOL

NASH, JOHN FORBES, JR. Enrolled 9/20/48 Department MATHEMATICS

Date and place of birth June 13, 1928 Bluefield, W.Va.

Single x Married

Bachelor and other degrees B.S. Carnegie Institute of Technology, June 1948; M.S. June 1948

Previous graduate study

Teaching experience

Address: Princeton B Pyne Tower, G.C.; 193 G.C.

Parent or guardian and address Mr. John F. Nash, 1405 Whitethorn St., Bluefield, West Virginia

1948-49 J.S.K. Fellow in Mathematics

1949-50 Atomic Energy Commission Predoctoral Fellow

A.M.; M.F.A. or M.S.E. granted

Address

PH.D.

French satisfactory May 27, 1949

German satisfactory March 4, 1949

General Examination Passed October 11, 1949

Dissertation subject

Dissertation accepted

Final Examination

Degree granted

Address

NASH, JOHN FORBES, JR. Department MATHEMATICS

FIRST TERM

1948-1949

537 Differential Equations	MacColl	P
501 Mathematical Logic	Church	P
525 Algebraic Topology	Steenrod	P

1949-1950

539 Partial Differential Equations	Schiffer	
Seminar on Theory of Games	Gale	
The Problem of Three Bodies and Related Questions (Institute for Advanced Study)	Siegel	

SECOND TERM

1948-1949

502 Mathematical Logic	Church	P
506 Algebra	Artin	P
526 Algebraic Topology	Steenrod	P

Most graduate courses occupy three hours weekly for a term. Full-year courses in Public Affairs (usually indicated by hyphenated numbers) occupy three hours weekly for two consecutive terms.

Explanation of marking system:

E - - - - - - - - - - -Excellent
VG - - - - - - - - - - Very Good
G - - - - - - - - - - -Good
P - - - - - - - - - - Passing
F - - - - - - - - - - Failed

Grades in parentheses are tentative mid-year grades in full-year courses. The final grade in a full-year course is the grade for the whole year.

Examiners Report — General Examination

Excellent

Examiners Report — Final Examination

CARNEGIE INSTITUTE OF TECHNOLOGY — OFFICE OF THE REGISTRAR — PITTSBURGH, PENNSYLVANIA

DAY / EVENING / GRADUATE / SUMMER / REGULAR / SPECIAL	Date of Birth	College	
	JUNE 13, 1928	ENGINEERING	First Name: JOHN — Middle Name: FORBES — Last Name: NASH, JR.
	Date of Admission	Department	Name of Parent or Guardian: MR. J. F. NASH — Address: 1405 WHITETHORN ST. BLUEFIELD, W. VA.
	JUNE 1, 1945	MATHEMATICS	Name of Preparatory School: BEAVER HIGH SCHOOL, BLUEFIELD, VA. COLLEGE — Address: BLUEFIELD, W. VA.

ENTRANCE RECORD

SUBJECTS	Initial Credit	Entrance Condition	Final Credit	SUBJECTS	Initial Credit	Entrance Condition	Final Credit	SUBJECTS	Initial Credit	Entrance Condition	Final Credit
English			4	General Science			1	French			
Algebra			1½	Natural Science			1	German			
Plane Geometry			1	History				Spanish			
Solid Geometry—Trigonometry			½ + ½	U. S. History and Civics			3				
Physics			1	Problems of Democracy							
Chemistry			1	Latin			2	Total			16½

COLLEGE RECORD

6-27-48 AWARDED B.S. Degree in Mathematics — 11th Honor Student. 6-27-48 Awarded M.S. Degree in Mathematics.

SUBJECTS	Subject Number 1st Sem.	Subject Number 2nd Sem.	Attend Hours Per Week	Units Per Sem.	9/5/45 - 1/29/46 1st Sem. Gr.	9/5/45 - 1/29/46 2nd Sem. Gr.	1946 Feb.-Sept. 1st Sem. Gr.	1946 Feb.-Sept. 2nd Sem. Gr.	Oct. '46 - May '47 1st Sem. Gr.	Oct. '46 - May '47 2nd Sem. Gr.	1947—1948 1st Sem. Gr.	1947—1948 2nd Sem. Gr.	19—19 1st Sem. Gr.	19—19 2nd Sem. Gr.	Total Sem. Hrs.
General Chemistry	E101	E102	6	9	A	A									
Mathematics	C1	C2	4	12	ad *	ad *									
English	C105	C106	2	6	ad *	ad *									
Social Relations	C701	C706	2	6	A	A									
~~Eng. Dwg. & Desc. Geom.~~	~~E001~~	~~E002~~	~~6~~	~~9~~											
Hygiene	H1		1	2		A									
Physical Education	H10	H11	2	0	P	P									
R. O. T. C.	R1	R2	3	5	B										
Physics Rec.	E91	E92	3	8	A	A									ad-36
Physics Lab	E91	E92	3	4	A	B									C.I.T. 61
															97 235 Clear
Mathematics	C3	C4	4	12	A	A									
English	C107		2	6	B										
Social Relations	C715	C716	3	9			B	A							
Physics	E93		6	12			A								
Quant. Anal. Rec.	E112		1	3			B								
Quant. Anal. Lab.	E112		6	6			C								
Seminar	E129	E130	1	1			A	C							
Qual. Analysis		E105	6	9		A									
Physical Meas. Rec.		E73	2	5			A								
Physical Meas. Lab.		E73	3	4			B								
Physical Education	H13	H14	3	0			P	P							
Eng. Dwg. + Desc. Geom.	E005a		5	5				C							
															94 329 Clear
Mathematics		C21	3	9		A									
Physical Chem.	E125		3	9			B								
Algebra	C23	C24	3	9			A	A							
Vector Analysis		C22	3	9			A								
Social Relations	C725	C728	3	9				A		A					
Physical Chem. Rec.		E126	3	6				C							
Physical Chem. Lab.		E126	6	6				B							
German	C251	C252	3	9					A	A					
Adv. Calculus	S255	S256	4	12					A	A					
Proj. Geometry	S257	S258	3	9					B	A					
Tensor Calculus	S281		3	9					A						
Theor. Mechanics	S261	S262	3	9					A	A					
															165 618 Clear
Chem. Option	C146		9					C							
Theory of Functions	GS301	302	3	12					A	A					
Complex Variable	GS303	304	3	12					A	A					
Adv. Calculus	GS319		4	12						A GS319					
Social Relations	C754	C757	3	9							A	A			
Modern Algebra	S273	S274	3	9							A	A			
Theor. of Relativity	GS365		3	12							A				
Quant. Mech.	GS469		3	12							A				
Function Space	GS373		3	12							A				
Seminar	S297	S298	1	3							A	A			
Geol. of Relativity		GS366	3	12								A			
Elect. Networks		GS377	3	12								A			
Quant. Mechanics		GS470	3	12											
															171 666 Clear
Total Sem. Hours															
Total Units per Semester					50	59	67	57	72	84	57	45	ad-36	C.I.T. 491	527
Q. P. Earned per Semester					189	232	231	174	279	336	228	180			1848
Q. P. Per Unit					3.78	3.93	3.45	3.05	3.88	4.00	4.00	4.00			3.76

The present status of the student is: Graduated June 27, 1948

The student is entitled to an honorable dismissal.

Date of issue: June 30, 1948 — E. K. Collins

Recorder — Registrar

THIS TRANSCRIPT NOT VALID UNLESS IT BEARS THE ORIGINAL SIGNATURE OF REGISTRAR OR RECORDER

EXPLANATION OF RATINGS

Previous to Sept. 1, 1933 — Grades below passing are:

- A—Excellent
- B—Good
- C—Satisfactory
- D—Fair
- E—Poor
- I—(Incomplete), otherwise satisfactory.
- F—(Failure, must fulfill any special requirements and must pass in a second examination.
- R—(Repeat).

After Sept. 1, 1933 — Grades below passing are:

- A—Distinguished
- B—Superior
- C—Average
- D—Passing
- I—(Incomplete), otherwise satisfactory.
- F—(Failure), must fulfill any special requirements and must pass in a second examination.
- R—(Repeat).

After Sept. 1, 1937 — Grades below passing are:

- A—Excellent
- B—Good
- C—Satisfactory
- D—Passing
- P—Passing (Physical Education only)
- R—Failure
- I—Incomplete
- X—Conditional Failure

Ad.—Credit granted for work completed at another institution.
Unit of Work: One hour per week per semester of the student's time, whether recitation, laboratory or home preparation.

Dear Sir:

I should like to learn about the possibility of my obtaining some kind of teaching or research position at your university in the mathematics department. Would you be so kind as to send me information concerning such positions? I am also applying for a fellowship but the application blanks for this do not include those other possibilities. If you would send me blanks with which to apply for such positions I should be very grateful to you.

I expect to graduate next June at the Carnegie Institute of Technology with the Master's and Bachelor's degrees in mathematics. My address is c/o the above school, Pittsburgh 13, Pennsylvania

Respectfully yours,

John F. Nash, Jr.

12/31/47 offl. sent. Eur. add. for own.

Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania
December 29, 1947

Dean of the Graduate School
Princeton University
Princeton, New Jersey

Dear Sir:

I shall graduate in June 1948 at Carnegie Institute of Technology with the Bachelor of Science Degree and the Master of Science Degree in Mathematics. Since I wish to continue graduate study in mathematics leading to the doctorate, I should like to apply for a fellowship and a teaching assistantship at your institution. I would appreciate it if you will kindly send me the necessary forms to be filled out and any additional information concerning fellowships and teaching assistantships.

Very truly yours,

John F. Nash, Jr.

John F. Nash, Jr.

JFN,Jr:VF

1/19/48. ack.

JAN 19 1948

CARNEGIE INSTITUTE OF TECHNOLOGY
SCHENLEY PARK
PITTSBURGH 13, PENNSYLVANIA

DEPARTMENT OF MATHEMATICS
COLLEGE OF ENGINEERING AND SCIENCE

January 15, 1948

Dean of Graduate School
Princeton University
Princeton, N. J.

Dear Sir:

It is a pleasure for me to support the application of Mr. John F. Nash, Jr. for a Fellowship or Scholarship. He has studied with me for three semesters, taking courses in Theoretical Mechanics, Tensor Calculus, and the Special Theory of Relativity. He expects his B.S. degree in June; at the same time he will qualify for the M.S. degree on account of graduate credits.

Mr. Nash is unique in my experience of students. I would rank him among the best I have had, and possibly he is the very best. At first impression, he might appear inferior, since he does not write out his work in polished form, nor does he lecture impressively. However, this external clumsiness is more than compensated by quickness of understanding, originality, and capacity for seeing the inner meaning of an argument, all unrivalled in my experience. While still a junior, he was capable of assimilating the most advanced work available in graduate lectures. On account of his speed of understanding he has been able to take many more courses than an ordinary undergraduate. Consequently his training is very wide, both in pure and applied mathematics.

He has not yet selected any special field of mathematical study. However, his inclination appears to be definitely in the direction of pure mathematics rather than applied, and therefore it is with our advice that he is seeking a fellowship or scholarship elsewhere, since our primary interest here is in applied mathematics.

From the standpoint of character and industry, Mr. Nash is all that can be desired. Also, he has a pleasant personality, and although stubborn in mathematical argument and full of confidence, his pre-eminence among the undergraduates here has not made him conceited.

Yours faithfully,

John L. Synge

John L. Synge, Head
Dept. of Mathematics

JLS:hl

JAN 26 1948

Ack. " " "

CARNEGIE INSTITUTE OF TECHNOLOGY

SCHENLEY PARK

PITTSBURGH 13, PENNSYLVANIA

DEPARTMENT OF MATHEMATICS
COLLEGE OF ENGINEERING AND SCIENCE

January 22, 1948

Dean of the Graduate School
Princeton University
Princeton, New Jersey

Dear Sir:

It is with the greatest pleasure that I second the application of Mr. John Forbes Nash, Jr. for a Fellowship or Scholarship to your institution for the academic year 1948-1949.

I have known Mr. Nash intimately for the past three years. He was a student of mine in several advanced courses in mathematics, and I consider him one of the most able students in mathematics in my entire experience of over thirty years of college teaching. In June, 1948, he will graduate from Carnegie Institute of Technology with the degrees Bachelor of Science and Master of Science in Mathematics. For a student to receive both the Bachelor's and Master's degrees at the same time is most unusual. Although his scholastic factor is not available at the moment, he should graduate at the head of his class. Since his field of study appears to be in the direction of pure mathematics rather than applied, it is with our advice and recommendation that he is applying for a fellowship or scholarship at some other institution. As you perhaps know, our primary interest at Carnegie Institute of Technology is in the field of applied mathematics.

Mr. Nash holds one of the most coveted scholarships on our campus, namely, the George Westinghouse Scholarship. In addition, he participated in the national William Lowell Putnam Mathematical Competition in 1946 and won an honorable mention. In 1947, he was a member of the Carnegie Institute of Technology team of three in this same competition and he placed among the ten highest contestants for which he won a cash award. The team won an honorable mention.

Mr. Nash is continually investigating original problems and gives promise of being a top-flight mathematician. He has published an article on Number Theory in the Carnegie Technical, and has written another article on Topology which will appear in a forthcoming issue. His main outside interest appears to be in the field of music.

Mr. Nash is a young man of exceptional ability and energy and is a very clear thinker on scientific problems. His training in mathematics, physics, and mechanics has been not only extensive but thorough. In my opinion, the institution that secures him as a graduate student in mathematics will be fortunate indeed. He has a pleasing personality and is co-operative at all times.

It is, therefore, with the greatest pleasure that I recommend Mr. John Forbes Nash, Jr. as an unusually able and well-trained young man. He should go far as an original investigator in the field of mathematics.

Very truly yours,

J. B. Rosenbach

J. B. Rosenbach
Professor and Assistant Head
Department of Mathematics

2/2/48 ack.

FEB 2 1948

CARNEGIE INSTITUTE OF TECHNOLOGY
SCHENLEY PARK
PITTSBURGH 13, PENNSYLVANIA

DEPARTMENT OF MATHEMATICS
COLLEGE OF ENGINEERING AND SCIENCE

January 30, 1948

Dean of the Graduate School
Princeton University
Princeton, New Jersey

Dear Sir:

This letter is written at the request of Mr. John F. Nash, who is applying for a fellowship in the Department of Mathematics at your institution.

I have known Mr. Nash for about three years, and have had him as a student in five different advanced courses in mathematics. He is probably the most able student with whom I have come in contact during twenty three years of teaching. He has a very keen mind, is quick to grasp new ideas, and is able to extend these ideas and obtain results in hitherto unexplored domains. He is far above all other students in native ability in those classes where I have had a chance to observe him; and from conversations with other members of the faculty at Carnegie Tech, I find that their opinions are about the same. I cannot speak too glowingly about this candidate's ability; if there is such a thing as a genius level, his ability reaches it. I have never before said this of any other student. There is no doubt in my mind that this young man will some day, if he has the opportunity to continue his studies, be one of the famous mathematicians of the country.

Incidentally, in the Lowell Putnam Mathematical Competition, which is open to undergraduate students of the United States and Canada, he placed among the first ten, even though he was just a third-year student at the time of the examination.

As far as I know his character is flawless, and he is emotionally stable. He has a pleasant personality; I would not say that he is shy, but he is almost shy. He is respectful and cooperative. He is aware of his ability, but it has not made him arrogant or caused him to feel superior to others.

In wishing to be honest and give as correct a picture as possible, I must mention one flaw. His ability to write and convey his ideas to others needs to be improved, and I feel certain that it can be improved. His mind is so quick, that in presenting material which others are to understand, he is apt to leave gaps in the written or oral explanations, which he is able to supply but which he thinks others have also done mentally as well as he has.

I feel certain that this boy will measure up to anyone else in scholastic ability and intellectual promise.

Yours very truly,

David Moskovitz

DM:HJ

David Moskovitz
Associate Professor of Mathematics

J. F. Nash, Jr.

Statement of Courses

I enclose pages from a catalogue describing some of my courses in mathematics. These are marked by black circles around their numbers. Any texts used have been indicated by the authors' names to the side of the corresponding course. In all of my mathematics courses I have had free opportunity for conferences with the instructor. But conferences were not scheduled.

Other courses I have taken which were not described in the catalogue are described below.

Freshman Mathematics ; Bluefield College, Bluefield, W. Va. Taken as a supplement to my high school work. Taught largely as a lecture course. Rosenbach and Whitman used for the algebra portion of the course. Graphs, liner systems, applications, equations in general, quadratics, applications, quadratic systems, progressions, use of induction, binomial theorem, rules of inequalities, complex numbers, theory of equations, synthetic division.

Brink - Plane and Spherical Trigonometry. Trigonometric functions, logarithms, identities, standard trigonometrical problems in the plane, some treatment of spherical problems.

I forget the name of the analytic geometry text used in this course. We studied straight lines, distance, perpindicular straight lines, oblique coordinates, circles, ellipses, parabolae, hyperbolae, transformations of axes, the quadratic discriminant and the general second degree equation, planes, spheres, the cone, hyperboloids, paraboloids, ellipsoids, tangents, normals, general methods for obtaining them. The three portions together comprised 12 semester hours, or 36 credit hours

Projective Geometry ; Winger was the text's author. The text was extremely non-rigorous, in my opinion, in the partly algebraic sections in particular. I thought the method of development could have been much improved. We studied the inverse conception of a curve, as a set of tangents, emphasized the dual points in both an algebraic and geometrical manner. To name chapters, we took up duality, the

J. F. Nash, Jr.

"line at infinity", projective properties and the double ratio, projective coordinates, the conic, collineations in one dimension. Largely a lecture type presentation.
I found myself in frequent conflict with the instructor.

Special Relativity; no text. Basic general Lorentz transformation developed in a rather axiomatic fashion. Mechanics and electromagnetic theory developed in a rather formal fashion. Physical applications treated at various points in the development of the theory. Lecture - Recitation.

"Function Space" and Applications. Von Neumann's "Mathematische Grundlagen der Quantenmechanik" used as a text. The intention of the course is to take up the material to page 101. That is, the predominantly mathematical section. Some of the early sections dealing with the physical conceptions were omitted. Largely student presentation.

Projective Geometry - 3 semester hours - 9 credit hours

Special Relativity - 3 semester hours - 12 credit hours

"Function" Space - 3 semester hours - 12 credit hours

J. F. Nash, Jr.

Text ↓

S-231. Statics. Analytic and graphic treatment of concurrent, non-concurrent, and parallel forces; couples; stresses in frames and trusses; stress diagram; centroids; moments and products of inertia; principal axes; centers of pressure; weighed flexible cords. Prereq.: To be admitted to S-231, students must have had (1) S-402; (2) S-223 (or S-228) or must take it concurrently. 3 hrs. rec., 5 hrs. prep., 8 units.

S-232. Dynamics. Rectilinear and curvilinear motion of a particle; relative velocities and accelerations; the work-energy principle; translation of a rigid body; rotation of a rigid body about a fixed axis; balancing of a shaft; plane motion; impact. Prereq., S-231. 3 hrs. rec., 5 hrs. prep., 8 units.

S-233. Strength of Materials. Analysis of stress and strain; shear, bending moment, stresses and deflection of beams; torsion; combined stresses; reinforced concrete beams; columns; theories of failure. Prereq., S-231. 3 hrs. rec., 5 hrs. prep., 8 units.

Rosenbach. Whitman

S-241 and S-242. Higher Algebra I and II. Development of the number system of algebra; theory of algebraic equations; fundamental theorem of algebra; determinants; symmetric functions; resultants, discriminants, and elimination; Graeffe's method; polynomials; linear dependence; systems of linear equations; matrices; linear transformations; invariants and covariants; bilinear and quadratic forms; elementary divisors; invariant factors. The relations of the algebraic theories to geometry will be emphasized throughout the course. Prereq.: To be admitted to S-241, students must have had S-223 (or S-227) or must take it concurrently. 3 hrs. rec., 6 hrs. prep., 9 units each semester.

Cankwright Bocher

S-250. Ordinary Differential Equations. Ordinary differential equations with applications to geometry and the physical sciences; singular solutions; operator methods; total differential equations; systems of equations; series solutions; approximate solutions. Prereq., S-228. 3 hrs. rec., 6 hrs. prep., 9 units.

Kells

S-251. Ordinary and Partial Differential Equations. Ordinary and partial differential equations, with applications to geometry and the physical sciences; singular solutions; operator methods; total differential equations; systems of equations; series solutions; approximate solutions of ordinary differential equations by graphical and numerical methods. Prereq., S-224. 3 hrs. rec., 6 hrs. prep., 9 units.

S-253 and S-254. Higher Mathematics for Science Students I and II. Vector analysis, Green's and Stokes' theorems; determinants, tensors, curvilinear coordinates; linear vector spaces, matrices, Hermitian spaces, eigenvalues, and eigenvectors; ordinary differential equations, solution in series, Legendre and associated Legendre polynomials, Bessel and Hankel functions, including elements of the theory of functions; Fourier series, partial differential equations by separation of variables, eigenvalue problems of classical physics; calculus of variations; elements of group theory and group representations. Throughout the course applications to the theory of molecular structure and quantum mechanics will be emphasized. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units each semester.

S-255 and S-256. Advanced Calculus I and II. A more critical study of the fundamentals of the differential and integral calculus than is possible in the elementary course. Topics include: Rolle's theorem, theorem of the mean, Taylor's theorem, and expansion of functions in power series; infinite series of constants and of functions; functions of several variables, and implicit functions; applications to curves and surfaces; introduction to vector methods; a more critical study of the definite integral; functions defined by definite integrals; improper integrals; multiple integrals; line, surface, and space integrals. Prereq., S-224 (or S-228). 4 hrs. rec., 8 hrs. prep., 12 units each semester.

Courant

Text ↓

S-257. Advanced Analytic Geometry. Algebraic representations and geometric interpretations in two and three dimensions; systems of curves and surfaces; transformations of coordinates and plane; general geometric theory of the equation of the second degree with one, two, and three variables; sections of a right circular cone; curves derived from physical conditions; algebraic and geometric solutions of equations; types of homogeneous representation in two and three dimensions. Prereq., S-241. 3 hrs. rec., 6 hrs. prep., 9 units.

Graustein

S-258. Differential Geometry. General properties of curves and surfaces; lines of curvature, conjugate and asymptotic lines, and geodesics; surfaces of center, surfaces of constant total curvature, ruled surfaces, and minimal surfaces; correspondences and mapping. Free use is made of the moving trihedral. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units.

S-259 and **S-260. Higher Mathematics for Engineering Students I and II.** Vector analysis, Green's and Stokes' theorems; linear algebraic equations, determinants, matrices; solution of transcendental and polynomial equations; ordinary linear differential and difference equations with applications to elementary network analysis and mechanical vibration theory; partial differential equations with a special study of the equation of diffusion, the wave equation, and Laplace's equation; Fourier series, Bessel functions, and Legendre polynomials; elements of the theory of functions of a complex variable. Throughout the course the theory will be illustrated by problems from the various fields of the engineering sciences. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units each semester.

Synge and Griffith

S-261 and **S-262. Theoretical Mechanics I and II.** A critical review of plane statics and dynamics with emphasis on the logical structure of the subject, including virtual work, friction, thin beams, cables, frames, projectiles, harmonic oscillators, planetary orbits, motion of a rigid body, stability, and impulsive motion; statics and dynamics in three dimensions with use of vectors, including the spherical pendulum, gyroscopes, Lagrange's equations; introduction to relativistic mechanics. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units each semester.

Phillips

S-263. Vector Analysis. Vectors and vector fields; scalar and vector products; gradient, divergence, curl; Stokes' theorem and divergence theorem; curvilinear coordinates; applications. Prereq., S-224 (or S-228) and S-403. 3 hrs. rec., 6 hrs. prep., 9 units.

S-264. Fourier Series. Study of Fourier series and integrals, with applications to solutions of boundary value problems in heat flow, vibrating strings and membranes, electric potential; introduction to Bessel functions and Legendre polynomials with applications. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units.

S-265. Theory of Potential. Attractions and potentials of solid, surface, and line distributions of mass or electricity; Gauss's theorem; equations of Laplace and Poisson; distribution of charge on conducting surfaces; systems of conductors; spherical harmonics. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units.

S-266. Hydromechanics. Pressure in a perfect fluid; equilibrium of the atmosphere; flotation and the stability of ships; mathematical description of plane fluid flow; incompressibility; irrotational motion; vortices; stream function; flow past a cylinder; Magnus force; equations of motion; Bernoulli's equation. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units.

S-271. Advanced Calculus III. Continuation of S-256 including theory of functions of a complex variable. Prereq., S-256 (or S-254, or S-260, and approval of instructor). 3 hrs. rec., 6 hrs. prep., 9 units.

including curvature, lines of curvature, geodesics, geodesic curvature, conjugate and asymptotic lines; surfaces of center, surfaces of constant total curvature, ruled surfaces, minimal surfaces, correspondences and mapping. Free use is made of the moving triedral. Reference text: Eisenhart's Differential Geometry. Prereq., C-4 and C-25; C-21, C-33, C-34 or C-57 are also desirable. Offered in alternate years. Offered in 1945-46. 12 units per semester.
PROFESSOR NEELLEY.

GC-7[illegible]a, b. REAL VARIABLE AND FOURIER'S SERIES. Real number system; point sets; sequences and series; limits and continuity; derivatives; the definite integral; improper integrals; infinite series; integrals with a parameter; Fourier's series and their convergence; Gibbs's phenomenon; Fourier's integrals. Prereq., C-33, C-34 or C-57; C-36 is also desirable. Offered in alternate years. Offered in 1944-45. 12 units per semester. PROFESSOR MOSKOVITZ.

Carslaw

GC-7[illegible]a, b. COMPLEX VARIABLE. Complex numbers; conformal representation; Cauchy's integral theorem; analytic continuation; Laurent's expansion; residues; entire functions and infinite products; meromorphic functions and partial fractions; multiple valued functions and Riemann's surface. Prereq., C-33, C-34 or C-57. Offered in alternate years. Offered in 1945-46. 12 units per semester. PROFESSOR MOSKOVITZ.

Townsend

GC-80a, b. READING AND RESEARCH. This course is designed to give opportunity to the graduate student in mathematics for reading and investigation not provided for in the regular courses. Each student admitted to the course will work in one of the following general fields and under the supervision of one of the indicated members of the department. Offered each year. 9 or 12 units per semester.

- Algebra, Professors Neelley and Rosenbach.
- Geometry, Professor Neelley.
- Analysis, Professors Rosenbach, Moskovitz, and Clippinger.
- Applied Mathematics, Professors Olds, T. L. Smith, Moskovitz, and Clippinger.

GC-81. THESIS. Units to be assigned.

NOTE: The advanced undergraduate courses are often acceptable for credit toward a graduate degree in other departments.

DEPARTMENT OF ENGLISH

PROFESSOR MCLEOD; ASSOCIATE PROFESSORS MACMILLAN, BEATTIE, AND WRIGHT; ASSISTANT PROFESSORS KIRKPATRICK AND PARSHALL*; MESSRS. GOODFELLOW, CROWELL, FREIMARCK*, AND HOLDEN*.

Unless otherwise indicated, each subject is offered each semester.

UNDERGRADUATE COURSES

C-105, C-106, and C-107. ENGLISH I. English composition; weekly themes; conferences; outside reading. 2 hrs. rec., 4 hrs. prep., 6 units each semester. ALL MEMBERS OF THE DEPARTMENT.

C-109 and C-110. ENGLISH I. English composition; weekly themes; conferences; outside reading. 3 hrs. rec., 6 hrs. prep., 9 units each semester.
ALL MEMBERS OF THE DEPARTMENT.

C-105, C-106, and C-107 or C-109 and C-110 are prerequisite for all higher courses in English.

*Leave of absence.

J. F. Nash, Jr.

C-28. Higher Mathematics for Science Students. Introduction to vector analysis, generalized coordinates, Lagrange's equations of motion, solution of boundary value problems in partial differential equations (problems in diffusion and heat flow, cooling, potential, and wave motion). Prereq., C-21. Second semester; 3 hrs. rec., 6 hrs. prep., 9 units.

C-33. Advanced Calculus I. A more critical study of the fundamentals of the differential calculus than is possible in the elementary course. Topics include: Rolle's theorem, theorem of the mean, Taylor's theorem, and expansion of functions in power series; infinite series of constants and of functions; functions of several variables, and implicit functions. Prereq., C-4. First semester; 3 hrs. rec., 6 hrs. prep., 9 units.

C-34. Advanced Calculus II. A more critical study of the definite integral; functions defined by definite integrals; improper integrals; multiple integrals; line, surface, and space integrals; a brief introduction to functions of a complex variable; and other selected topics varying from year to year. Prereq., C-33. Second semester; 3 hrs. rec., 6 hrs. prep., 9 units.

C-36. Fourier's Series. Study of trigonometric series, with applications to heat flow, electric fields, and other problems. Prereq., C-4. Second semester; 3 hrs. rec., 6 hrs. prep., 9 units.

C-55, C-56, C-57 (See Bulletin of Evening Courses.)

GRADUATE COURSES

Of the courses listed below, GC-80 will be available each year for properly qualified graduate students in mathematics, and the others will be given in accordance with the indicated scheme of alternatives when the demand justifies.

GC-58a, b. Heaviside Operational Calculus. A treatment of the subject, with applications to electrical networks and transmission lines and to mechanical problems. Justification of the methods by complex integration and by the Laplace Transformation. Prereq., C-21 or C-56, C-33 or C-57, or equivalent work in differential equations and in theory of functions of a complex variable. Offered in alternate years. Offered in 1945-46. 12 units per semester.
Professor T. L. Smith.

GC-59a, b. Tensor Analysis and the Theory of Relativity. The Lorentz Transformation and the special theory of relativity; Riemannian geometry and the general theory of relativity. Prereq., C-21 or C-56, C-22 or GE-91; C-33 or C-57, and E-82 are also desirable. Not offered in 1944-45. 12 units per semester.
Professor T. L. Smith.

GC-60a, b. Partial Differential Equations of Mathematical Physics. Boundary value and Eigenwert problems, orthogonal functions, vibrations and resonance, integral equations, and calculus of variations. Prereq., C-21, and preferably C-33, C-34 or C-57. This course is primarily for graduate students; undergraduates will be admitted only if they have taken or are taking C-33 and C-34. Offered in alternate years. Offered in 1944-45. 12 units per semester.
Professor T. L. Smith.

GC-70a, b. Modern Higher Algebra. Among the topics treated are fundamental properties of polynomials, determinants, and matrices; systems of linear equations, linear transformations, invariants, covariants; bilinear and quadratic forms; elementary divisors; theory of elimination; groups, rings, and fields. Reference texts: Bocher's Higher Algebra, Albert's Modern Higher Algebra, Van der Waerden's Moderne Algebra. Prereq., C-4, C-23, and C-25; C-33, C-34 or C-57 are also desirable. Offered in alternate years. Offered in 1944-45. 12 units per semester.
Professor Neelley.

GC-71a, b. Differential Geometry. Among the topics treated are general properties of curves including curvature, torsion; general properties of surfaces

SUBJECTS OF INSTRUCTION

The following is a description of the subjects of instruction offered by the various departments of the College of Engineering and Science and by the other colleges of the Institute that give instruction to the students in the College of Engineering and Science. The material is grouped as follows:

COLLEGE OF ENGINEERING AND SCIENCE

UNDERGRADUATE SUBJECTS

GRADUATE SUBJECTS

DIVISION OF HUMANISTIC AND SOCIAL STUDIES

DIVISION OF PHYSICAL WELFARE

DEPARTMENT OF MILITARY SCIENCE AND TACTICS

J. F. Nash Jr.

DEPARTMENT OF MATHEMATICS

PROFESSORS SYNGE, ROSENBACH, AND NEELLEY; ASSOCIATE PROFESSORS DUFFIN, HEINS, HOOVER, JOHNSON, LIGHTCAP, MOSKOVITZ, OLDS, SAIBEL, STARR, WEINSTEIN, AND WHITMAN; ASSISTANT PROFESSOR DIAZ; INSTRUCTORS DUROUX, EPSTEIN, GUSTAFSON, HAILPERIN, HAVEN, LAMBING, LEAMAN, McCURDY, PEACH, PEPPER, SACKS, SCHILD, SCHWARTZ, SLOAN, WALKER, WHIDDEN, AND ZORA.

UNDERGRADUATE COURSES

S-221. College Algebra and Trigonometry. Review of certain topics in entrance algebra; additional selected topics in college algebra; the trigonometric functions of any angle; the solution of triangles, both with and without the use of logarithms; radian measure; analytic trigonometry; trigonometric identities and equations; inverse trigonometric functions. 4 hrs. rec., 8 hrs. prep., 12 units.

S-222. Analytic Geometry and Calculus I. Fundamental notions of calculus; graphical representation; differentiation of algebraic functions in one variable, with applications to problems in geometry and mechanics such as maxima and minima, related rates, velocity, and acceleration; single and double integration of polynomials with applications to area, volume, and fluid pressure; analytic geometry of the straight line and the conic sections; parametric equations. Prereq., S-221; but it is taken by certain selected entering students who have had satisfactory training in the material of S-221 and whose other qualifications are acceptable. 4 hrs. rec., 8 hrs. prep., 12 units.

S-223. Analytic Geometry and Calculus II. Graphical representation and differentiation of trigonometric, inverse trigonometric, exponential, and logarithmic functions with applications; polar coordinates; integration by substitution, by parts, and by partial fractions; further applications of integration to mean value, work, and arc length; solid analytic geometry; double and triple integration with applications to area, volume, area of a surface of revolution, center of gravity, moment of inertia, and attraction. Prereq., S-222. 4 hrs. rec., 7 hrs. prep., 11 units.

S-224. Analytic Geometry and Calculus III. Introduction to statistical methods; Maclaurin and Taylor series; approximate integration; hyperbolic functions; partial differentiation with applications; ordinary differential equations of the first and second orders, linear differential equations with constant coefficients, applications. Prereq., S-223. 4 hrs. rec., 7 hrs. prep., 11 units.

Text ↓ Neely and Tracey

S-226. Plane and Solid Analytic Geometry. The analytic geometry of two and three dimensions, with emphasis on the *method* of using analysis in the study of geometry and geometry as a means of interpreting analysis. Prereq., S-221. 4 hrs. rec., 8 hrs. prep., 12 units.

S-227. Calculus I. This course is devoted largely to the differential calculus, including the fundamental notions of derivative and differential of a function of one variable, partial and directional derivatives of a function of more than one variable, with applications to tangents and normals, maxima and minima, rates, curvature, approximations and errors. The last part of the course includes the inverse problem of indefinite integration. Prereq., S-226. 4 hrs. rec., 8 hrs. prep., 12 units.

S-228. Calculus II. The topics treated include: the definite integral, both simple and multiple, with applications to area, arc length, volume, moment of inertia, fluid pressure, and work; infinite series and methods of approximation; hyperbolic functions; introduction to ordinary differential equations. Prereq., S-227. 4 hrs. rec., 8 hrs. prep., 12 units.

ference; modern methods of solving the problems of specification, distribution, and estimation. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units each semester.

S-295 and **S-296. Mathematical Statistics I and II.** Introduction to the principles and methods applicable to the problems of collection, analysis, and interpretation of observational data; measures of central tendency, dispersion, skewness, and kurtosis; Bernoulli, Poisson, and Gaussian distributions; Pearson curves and Gram-Charlier series; simple and multiple correlation and regression; relation of theory of probability to statistical inference; sampling theory, estimation, and tests of significance. Prereq., S-224 (or S-228). 3 hrs. rec., 6 hrs. prep., 9 units each semester.

S-297 and **S-298. Seminar in Mathematics I and II.** Recent papers and special problems are discussed at weekly meetings. Graduate students and seniors specializing in mathematics are required to attend regularly. Each student is required to lead the discussion at least once in each semester. 3 units each semester.

DEPARTMENT OF PHYSICS

PROFESSORS SEITZ, ESTERMANN; ASSOCIATE PROFESSORS CORBEN, CREUTZ, PRINE, PUGH, MAURER, SIMPSON; ASSISTANT PROFESSORS BESSEY, BOREMAN, FOX, KOEHLER, MICHENER, SUTTON, WILLIAMSON; INSTRUCTORS HAMMOND, LEIVO, VON HEINE-GELDERN.

UNDERGRADUATE COURSES

S-401. Physics I. Introduction to Statics: The concepts of Force and Torque, Equilibrium conditions, Hydrostatics, Archimedes' Principle. Prereq., S-221 (or concurrently). 2 hrs. rec., per week, 1 hr. lecture, and 2 hrs. laboratory in alternate weeks, 3½ hrs. prep., per week, 7 units.

S-402. Physics II. Introduction to Dynamics: Displacement, Velocity and Acceleration. Newton's laws of Motion and their application. Prereq., S-222 (or concurrently) and S-401. 2 hrs. rec. per week, 1 hr. lecture and 2 hrs. lab. in alternate weeks, 3½ hrs. prep. per week, 7 units.

S-403. Physics III. Heat, Sound and Light. Temperature, thermal expansion, Equation of State. Calorimetry. Wave motion and fundamental properties of Sound waves. Photometry, Reflection and refraction of light. Wave optics: interference and diffraction. Prereq., S-223 (or concurrently) and S-402. 2 hrs. rec., 1 hr. lec., 3 hrs. lab., 5 hrs. prep., 11 units.

S-404. Physics IV. Electricity and Magnetism. Electrostatics, Production and effects of electric currents. Measurement of electric quantities. Fundamental Properties of Magnetism. Prereq., S-223 (or concurrently) and S-402. 2 hrs. rec., 1 hr. lec., 3 hrs. lab., 5 hrs. prep., 11 units.

S-405. Physics A. Mechanics, Heat and Sound. An accelerated course in elementary Physics. For advanced standing students only. Prereq., S-223 (or concurrently). 1 hr. lec., 3 hrs. rec., 3 hrs. lab., 8 hrs. prep. per week, 15 units.

S-406. Physics B. Light and electricity. Continuation of S-405. Prereq., S-223.

S-407. Survey of Elementary Physics. A concentrated course for advanced standing students who require additional background. Prereq., S-223 (or concurrently). 1 hr. lec., 3 hrs. rec., 8 hrs. prep., 12 units.

S-409. Physical Measurements. Basic principles in physics supplementing the courses in General Physics. Also a critical study of methods of making,

J. F. Nash Jr.

S-272. Special Functions. Definition, properties, and applications of the more common non-elementary functions, including the following: Gamma and Beta functions; Bessel functions; Legendre, Hermite, Laguerre, and Tchebychef polynomials; elliptic functions; hypergeometric function. Prereq., S-271. 3 hrs. rec., 6 hrs. prep., 9 units.

S-273 and **S-274. Modern Algebra I and II.** Integers, rational numbers and fields, and real numbers; polynomials; complex numbers; group theory; vectors and vector spaces; the algebra of matrices; linear groups; rank and determinants; algebra of classes; transfinite arithmetic; rings and ideals; algebraic number fields; Galois theory. Prereq., S-242. 3 hrs. rec., 6 hrs. prep., 9 units each semester. Birkhoff and MacLane

S-276. History of Mathematics. The development of mathematics through the calculus, emphasizing significant events, eminent mathematicians, the related literature, and sources of our information. Prereq., S-224 (or S-228). 2 hrs. rec., 4 hrs. prep., 6 units.

S-277 and **S-278. Practice Teaching I and II.** Classroom teaching under supervision; reading of pedagogical literature relating to mathematics; reports; conferences. Prereq., S-224 (or S-228). Number of units variable, depending on amount of work done.

S-281. Tensor Calculus and Applications. The tensor concept; absolute and covariant differentiation; geodesics; curved spaces and the curvature tensor; Cartesian tensors; Lagrange's dynamical equations; fundamental equations of continuous media and electromagnetic radiation; introduction to general relativity. Prereq., S-256 (or S-254, or S-260). 3 hrs. rec., 6 hrs. prep., 9 units. no text

S-284. Theory of Vibrations. Vibrations of a discrete system treated by Lagrange's equations; normal modes and frequencies; dissipative systems; forced vibrations; vibrations of elastic bars and membranes; sound waves; methods of approximating natural frequencies. Prereq., S-261. 3 hrs. rec., 6 hrs. prep., 9 units.

S-287. Elasticity. Analysis of strain and stress; equations of equilibrium; stress-strain relations in isotropic bodies; equations of compatibility; bending of beams, torsion, and flexure; irrotational strain; spherical shell and hollow cylinder under pressure; plane strain and generalized plane stress. Prereq., S-251. 3 hrs. rec., 6 hrs. prep., 9 units.

S-288. Aerodynamics. General three-dimensional motion of a fluid; vortex tubes; irrotational motion; flow past a sphere; plane flow patterns discussed with complex variable; theorems of Blasius and Joukowsky; conformal transformation; Joukowsky and other profiles; airfoil of finite span and induced drag. Prereq., S-266. 3 hrs. rec., 6 hrs. prep., 9 units.

S-291 and **S-292. Statistical Quality Control I and II.** Elementary statistical methods and their application to industrial problems; construction and interpretation of Shewhart charts; Sealy's modified techniques; Dodge-Romig and Army Ordnance Tables for acceptance sampling; quality assurance for sampling by measurement; introduction to sequential analysis; methods of correlation; elementary analysis of variance. This course is designed primarily to fill the needs of persons responsible for the design, control, or appraisal of manufacturing processes or products. Prereq., college algebra, industrial experience, and approval of instructor. 3 hrs. rec., 6 hrs. prep., 9 units each semester.

S-293 and **S-294. Introduction to Probability and Mathematical Statistics I and II.** The theory of probability and its application to the problems of collection, treatment, and interpretation of statistical data; measures of central tendency, dispersion, and correlation; sampling techniques; statistical in-

April 1,1948

My dear Mr.Nash:

I have pleasure in informing you that you have been appointed by the Faculty of Princeton University as

J.S.K.FELLOW

for the academic year 1948-1949, at a stipend of $700. Notice of acceptance before April 15th is requested. The academic year will begin on September 20,1948, and graduate students are required to register on or before that day.

Sincerely yours,

Mr.John Forbes Nash,Jr.

PRINCETON UNIVERSITY

THE GRADUATE SCHOOL

This is to certify that on March 4,1949

Mr. John F. Nash, Jr.

was tested in his reading knowledge of German
(Language)

and that he satisfactorily sustained this test.

(Signed) E. Artin

For the Department of Mathematics

PRINCETON UNIVERSITY

THE GRADUATE SCHOOL

This is to certify that on May 27, 1949

Mr. John F. Nash, Jr.

was tested in his reading knowledge of French
(Language)

and that he satisfactorily sustained this test.

(Signed) [signature]

For the Department of Mathematics

PRINCETON UNIVERSITY

THE GRADUATE SCHOOL

Date **October 12,1949**

Mr. **John F. Nash, Jr.** took the General Examination given by the Department of **Mathematics** on **October 11, 1949** (date). In the judgment of the Department he has sustained the examination.* His principal examiners were: **Professors N. E. Steenrod, Chairman, Alonzo Church and D. C. Spencer**

Estimate of the quality of performance of the candidate:

The candidate passed an excellent examination.

Rating:

- Excellent x
- Very Good
- Good
- Passing
- Failed

Emil Artin

Departmental Representative

*If the candidate did not sustain the examination, the Dean of the Graduate School should be so informed by letter.

A. E. C. Fellow

193

APR 13 1949

Dear Sir,

I wish to apply for reassignment to the room that I now occupy. This is PT B. As a second choice, another room in this group, and as third choice a room in group III S.

Sincerely yours,

John F. Nash, Jr.

My dear sir:

Since there seems to be no possibility of any other assignment I am writing to give my acceptance of the room assignment offerred me. I regret that no consideration was given to my own expressed wishes in this affair. In connection with policy on fellowship holders I find it interesting to note:

(1) I am not on a university fellowship

(2) Room assignments for 1949-50 having been made before all Proctor fellowships were awarded, I imagine the next term will find some Proctor fellows assigned to rooms quite unlike the traditional Proctor Fellow assignments.

Very sincerely yours,

John F. Nash, Jr.

June 8,1949

Dear Mr.Nash:

I acknowledge receipt of your recent undated letter,accepting a room assignment in the Graduate College for next year. Please be assured that there is no compulsion connected with your accepting an assignment. It would appear,from the tone of your communication,that you might prefer to live elsewhere. If that is the case, I will be happy to assign your suite to a person on the long waiting list.

Very sincerely yours,

Dean of the Graduate School

Mr.John Nash,
Graduate College,
Princeton,New Jersey

OCT 11 1949

Dear Dean Thorpe,

When I first spoke with you about my refrigerator I did not know that there was no basement under the 19th entry. And I have recently observed that there is a refrigerator kept in room 91. For these reasons I wish to inquire about moving the refrigerator into my room.

I have been trying to sell it but I am coming to beleive that I can best hope to sell it next summer or next academic year. Therefore I should like to get some use from it and the only really satisfactory location of it for that purpose is in my room.

Yours Sincerely,

John J. Mock, Jr.

Nash

Abstract

This paper introduces the concept of a non-cooperative game and develops methods for the mathematical analysis of such games. The games considered are n-person games represented by means of pure strategies and pay-off functions defined for the combinations of pure strategies.

The distinction between cooperative and non-cooperative games is unrelated to the mathematical description by means of pure strategies and pay-off functions of a game. Rather, it depends on the possibility or impossibility of coalitions, communication, and side-payments.

The concepts of an equilibrium point, a solution, a strong solution, a sub-solution, and values are introduced by mathematical definitions. And in later sections the interpretation of these concepts in non-cooperative games is discussed.

The main mathematical result is the proof of the existence in any game of at least one equilibrium point. Other results concern the geometrical structure of the set of equilibrium points of a game with a solution, the geometry of sub-solutions, and the existence of a symmetrical equilibrium point in a symmetrical game.

As an illustration of the possibilities for application a treatment of a simple three-man poker model is included.

File

Dean Taylor:

On this statement for Nash, do you want me to put under 6. that he received the rating of excellent on his final examination for the Ph.D. ?

I wrote this in & mailed

MCF

HST

July 5, 1950

FR[illegible]
HUGH S. TAYLOR

Dept of Mathematics

Please supply the desired information and return to me for signature

HST

The RAND Corporation
1500 FOURTH ST • SANTA MONICA • CALIFORNIA

June 26, 1950 L-5466

Princeton University
Princeton
New Jersey

Attention: Registrar

Gentlemen:

The RAND Corporation is conducting research for the United States Air Force on matters affecting the national security. In order that we be in a position to recommend our personnel for security clearance, it is necessary for us to verify certain factual data furnished by our employes and to request information concerning their character.

Dr. John F. Nash, Jr., who is retained as a consultant by The RAND Corporation, was a former student at Princeton University. We should greatly appreciate your answering the enclosed questionnaire concerning Dr. Nash and returning it in the enclosed stamped envelope.

You may be assured the information you furnish will be treated in utmost confidence.

Very truly yours,

Cecil A. Weihe
Personnel Manager

CAW:jd

Enclosure

Date **May 31, 1950**

Mr. **John Nash** was examined orally by the Department of **Mathematics** on **May 29, 1950**. In the judgment of the Department he sustained the examination, and is hereby recommended for the degree of Doctor of Philosophy.*

His principal examiners were **Professors J. W. Tukey (Chairman), E. Artin, and David Gale**

Estimate of the quality of performance of the candidate:

The candidate passed an excellent examination in the field of his thesis.

Final Rating:
- Excellent **x**
- Very Good
- Good
- Passing
- Failed

Departmental Representative

*If the candidate did not sustain the examination, the Dean of the Graduate School should be so informed by letter.

PRINCETON UNIVERSITY
OFFICE OF THE CONTROLLER

№ 248

DATE 3/21/50

RECEIVED FROM Nash, John F. $ 40.00

FOR

CREDIT DISTRIBUTION

Grad Fee	I-23	15	00
PHD Deposit	E 1621	25	00

G. A. MILLS, *Controller*

5M—11-49—Form 3

BY J.M.D.

Date May 22, 1950

Mr. John Nash presented to the Department of Mathematics in candidacy for the degree of Doctor of Philosophy a dissertation with the title:

Non-Cooperative Games

Written under the direction of Professor A. W. Tucker

The Department appointed Professors Tucker and Tukey to read this dissertation and, after consideration of their report, the Department hereby recommends to the Faculty the acceptance of the dissertation. Estimate of the dissertation:

This is a highly original and important contribution to the Theory of Games. It develops notions and properties of "non-cooperative" finite n-person games which are very interesting in themselves and which may open up many hitherto untouched problems that lie beyond the zero-zum two-person games. Both in conception and in execution the thesis is entirely the author's own.

The Department desires to hold the Oral Examination of the candidate on Monday, May 29 at 10 Am in Fine 207.

Chairman or Departmental Representative

Before the Oral Examination can be authorized, it will be necessary for the candidate to file two copies of his dissertation in the Office of the Graduate School, and also a receipt from the Controller of the University for the payment of the diploma fee of $15. and the $25. deposit to ensure the publication of the dissertation. The Department is requested to notify the candidate of these conditions.

If these conditions have been met and the desired date is satisfactory, the Department will be notified that it is authorized to hold the examination.

The Graduate School

Professor E.Artin

The Graduate School

Final oral examination — May 25,1950

Dear Mr.Artin:

Your Department is hereby authorized to conduct the final oral examination of Mr.John Nash at 10 o'clock Monday morning,May 29,1950 in Room 207 Fine Hall. Please see that each member of your Department is notified of this examination, and that a notice concerning it is placed on your departmental bulletin board.

Very sincerely yours,

FURTHER QUESTIONS TO BE ANSWERED BY EVERY APPLICANT

Single? ..✓...; Married? ......, Number of children?......; (If single, do you expect to change this status before the end of the next academic year?) ....no........

If married, or expect to be married before the end of the next academic year, have you additional resources to provide for the support of your family next year? ....

Can you read books in your chosen field of study in French? ..no.......... In German? ......yes....

Can you read simple Latin prose? ...no........

What foreign languages other than these can you read? ..none........

Since a reading knowledge of French and German is necessary for graduate study as conducted at Princeton for the degrees of M.A. and Ph.D., it is important that a candidate acquire such a reading knowledge as quickly as possible. Such candidates are expected to take an examination in one of these languages early in the first term. (See catalogue for special departmental regulations concerning language examinations.) If you are applying as a candidate for an M.A. or Ph.D. degree and are not prepared to take an examination in these languages upon admission, what plan have you for increasing your attainment in these languages before next year? .I plan either to improve my German and learn French this summer or next Fall by reading the literature.

Indicate original work or investigation which you have done, and list with title, date, and place of publication any books or contributions to periodicals.

Co-author of article in Oct. 1945 Electrical Engineering [work of a simple nature, done while in High School.] While in college I have worked on several problems of my own invention in number theory, topology, economic theory, and analysis.

Indicate, with dates, academic positions you have held, such as fellowships and teaching positions, giving the name of the institution:

..........none.

Indicate, with dates, business positions you have held, giving the name of the firm:

..Summer 1947 Student Trainee at Westinghouse Research Laboratory [because I had a G. Westinghouse Scholarship]

To what career do you look forward? Research in Math. and its application to allied fields..

How many years of graduate study do you now contemplate? no more than 2 before Ph.D.....

Check degree for which you plan to qualify. M.A. .... M.F.A. .... M.S.Eng. .... Ph.D. ..✓....

Signature of applicant: John F. Nash, Jr.

Check below the items you are supplying:

Transcript of record: undergraduate ......✓...... graduate ..✓....

Graduate Record Examination .... Photographs ..✓....

Detailed Statement of Courses ..✓....✓.. Other Documents ....

When the above questions have been answered, your application should be forwarded to the Dean of the Graduate School, Princeton University, Princeton, New Jersey.

Please consider me for a teaching position also.

Published daily since 1892

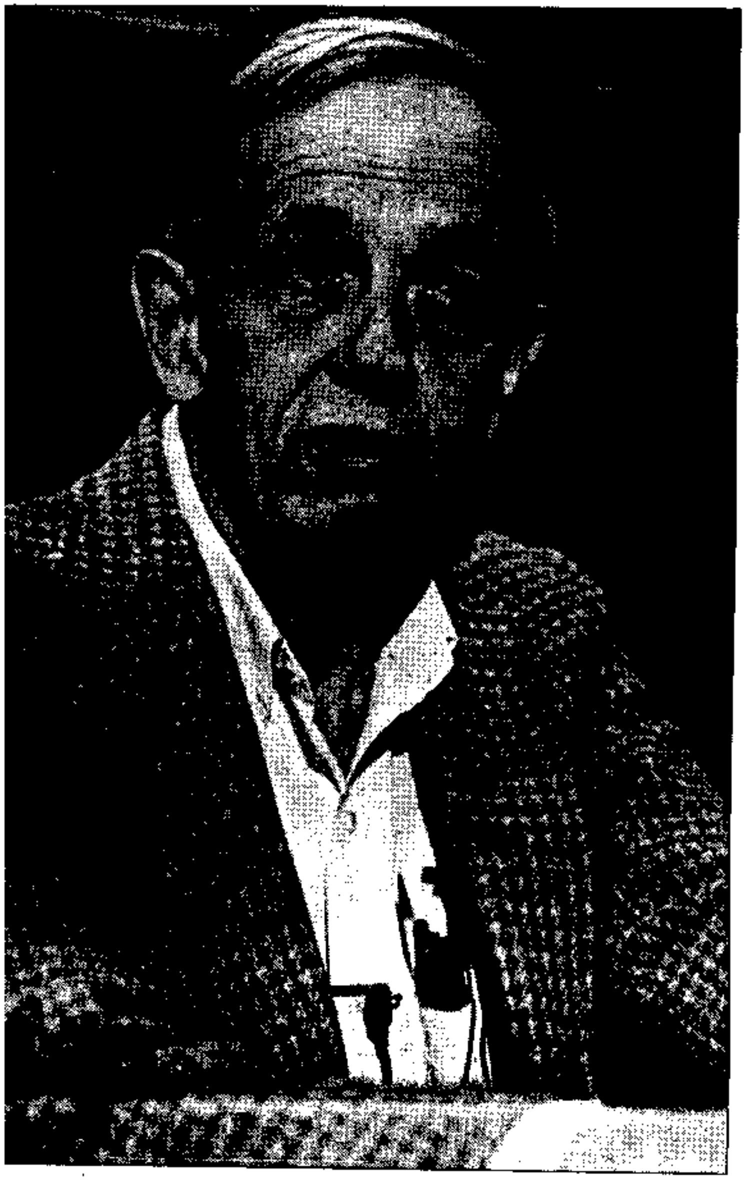
The Daily PRINCETONIAN

Today's weather: Sunny Low 60s

Vol. CXVIII, No. 98 | Princeton, New Jersey, Wednesday, October 12, 1994 | ©1994 | 30 Cents

Nash wins Nobel Prize in economics

Nobel committee cites classic work in game theory



By MELISSA SCHAPIRA

Continuing its recent recognition of Princeton's faculty, the Royal Swedish Academy of Sciences announced yesterday that Visiting Research Collaborator John F. Nash GS '50 was one of three recipients of the 1994 Nobel Prize in Economic Sciences.

Reporters, photographers and television crews joined students and faculty in a packed Jadwin Hall auditorium for a press conference following the announcement.

Answering questions from the crowd, Nash said he felt the award was "a great honor." However, he quipped, "the money could be better."

Money

Nash will share the $930,000 prize with John Harsanyi, a retired professor from the University of California at Berkeley, and Reinhard Selten from the University of Bonn.

He declined comment on how he will use the award money, joking that a response "might affect his credit rating."

Explaining that he had heard rumors of his selection for the award as early as Monday, Nash said he awoke early yesterday in anticipation of the "traditional phone call" to notify Nobel winners.

"I should have stayed in bed," he quipped, noting that the call, which he had expected between 6 a.m. and 7 a.m., did not come until after 7 a.m.

Nash equilibrium

Nash is best known for the concept of the "Nash equilibrium," which he presented in his Ph.D. thesis for the university's mathe-

(Continued on page four)

Nash's work helps predict decision-making process

By MELISSA SCHAPIRA

While most students in microeconomics courses are familiar with the "Prisoner's Dilemma," few may realize that the relatively simple model is an example of the theory that just earned John Nash GS '50 a Nobel Prize.

Nash is best known for introducing the concept of equilibrium in competitive games. In his Ph.D. thesis, Nash proposed his theorem that "every finite game has an equilibrium point."

Assistant economics professor Timothy Vanzandt explained game theory as "the strategic interaction between people (who)

(Continued on page five)

Police discover dead man in van, say death due to natural causes

By HOWARD GERTLER

Authorities yesterday identified the man found dead in a van on Western Way Monday night as Richard Hagadorn, a local painter.

An autopsy performed yesterday

white T-shirt and was surrounded by clothes and boxes, he said.

James then called for back-up. When the additional officers arrived, Best said, they "realized he'd apparently expired."

searching and photographing the inside of the van.

"Earlier in the evening, police had seen (Hagadorn) alive in the vehicle — around five o'clock," Slaboda said. The actual time of

Nash collects Nobel for theories

(Continued from page one)

matics department in 1950. His theorem describes a situation in which all players in a game make optimal decisions when they have incomplete knowledge of their competitors' choices.

While his work has been influential in the field of economics, Nash said his interests lie primarily in mathematics and theoretical physics. He noted that other disciplines, such as psychology, have "certainly been relevant" in the study of game theory, but questioned whether "a social science (is) really a science." Economics, he said, is "on the borderline."

When asked how his work had changed that particular discipline, he remarked, "I see my name cited more often."

Mathematics department chair Joseph Kohn praised Nash as a "superb mathematician." Alluding to a quote from Isaac Newton, he noted that "a whole generation of mathematicians has seen further by having stood on the shoulders of John Nash."

Nash joins a large contingent of previous Nobel winners associated with the university. Last year, three members of the university community were recognized for their accomplishments.

In October 1993, humanities professor Toni Morrison earned the Nobel Prize for Literature. Morrison, author of the Pulitzer Prize-winning novel "Beloved," was the first African-American woman to receive the award.

Six days after the announcement of Morrison's prize, the Swedish Academy awarded the Nobel for physics to Princeton Plasma Physics Laboratory researcher Russell Hulse and professor Joseph Taylor.

Hulse and Taylor shared the prize for their discovery of the first binary pulsar. Their research, which began nearly 20 years before they received the Nobel, confirmed important predictions of Einstein's general theory of relativity.

Math research

Although no Nobel Prize specifically recognizes accomplishments in mathematics, Nash is not the only Princetonian whose work in that field has recently received attention. In 1993, mathematics professor Andrew Wiles announced that he had found a proof for Fermat's Last Theorem, a problem that had daunted mathematicians for

(Continued on page seven)

Princeton's Nobel Prize Winners

Year Of Award	Name
Economics	
1979	SIR W. ARTHUR LEWIS, economics professor emeritus
1994	*JOHN NASH GS '50, visiting research collaborator
Physics	
1921	ALBERT EINSTEIN, Institute for Advanced Study
1927	ARTHUR COMPTON GS '16
1928	OWEN RICHARDSON, physics professor
1933	P.A.M. DIRAC, visiting professor in mathematics and physics
1937	CLINTON DAVISSON GS '11
1945	WOLFGANG PAULI, physics lecturer
1956	JOHN BARDEEN GS '36
1961	ROBERT HOFSTADTER GS '38
1963	EUGENE WIGNER, physics professor
1965	RICHARD FEYNMANN GS '36
1972	JOHN BARDEEN GS '36
1977	*PHILIP ANDERSON GS '49, physics professor
1978	ARNO PENZIAS, visiting lecturer
1979	STEVEN WEINBERT GS '53
1980	*VAL FITCH, physics professor emeritus

Game theory

(Continued from page five)

for him, depending on others' strategic choices.

Dixit said Nash equilibrium has been applied in the determination of fiscal and monetary policy. The Treasury and Congress, who determine fiscal policy, can be considered one "player" in the "game." The Federal Reserve, which sets monetary policy, is a second player with a "slightly different strategy," he explained.

Economics professor Richard Quandt described Nash equilibrium as "a state in which, given what the other players are doing, what any one player is doing is optimal."

"It's essentially a mathematical concept," he said, adding that the theory has "zillions of applications in game theory."

The Prisoner's Dilemma, created by Nash's graduate thesis adviser, mathematics professor Albert Tucker, is an example of a game that can be solved using Nash equilibrium.

The Prisoner's Dilemma is a theoretical situation where two parties are forced to compete rather than cooperate to reach an optimal solution.

OFFICIAL NOTICES

The Daily Princetonian publishes notices as a service to the university community. Notices will NOT be printed unless they are submitted by 1 p.m. the day before they appear. Each submission will run for a maximum of THREE DAYS.

FELLOWSHIPS/FOREIGN STUDY

Classes of '96, '97 and '98: National Endowment for the Humanities Younger Scholars Award. The award provides funding for noncredit-independent summer research and writing projects in the Humanities. Pick up an application now in 408 West College. The application deadline is Nov. 1, 1994. (12)

Whitaker Foundation Graduate fellowships in Biomedical Engineering: For seniors who intend to work towards a Ph.D. or Sc.D. degree in engineering, with a concentration in biomedical engineering. Provides a $16,000 living allowance, plus a cost-of-education allowance paid directly to the study institution in lieu of tuition and fees, and an additional payment of $1,500 to the institution to be used for the professional development of the fellow. More information is in 408 West College. Deadline: December 9, 1994. (12)

Peterhouse — Cambridge Research Studentships. A maximum of three awards will be given to students who intend to pursue a Ph.D. at the University of Cambridge. Provides payment of university fees, plus an allowance for living expenses. Note that you must apply separately for admission to Cambridge to be considered. More information is available in 408 West College. Deadline for applications: April 1, 1995. (12)

Study Abroad at King's College, University of London. An informational meeting will be held on Thurs., Oct. 13 at 4 p.m. in 301 West College. Call Dean Kanach's office at 8-5524 if you are unable to attend. (12)

Labouisses Fellowship — one year foreign research/service grant for graduating senior interested in problems bearing on improvement of conditions in the less developed world. Informational meeting Fri., Oct. 14, at 12 p.m. in Room 215 Bendheim Hall. Applications available in Room 117 Bendheim Hall, 8-4852. If questions contact John Waterbury, 8-4850. (14)

CAREER SERVICES

Attention freshmen, sophomores, and juniors! Just a reminder that the on-campus recruiting program is not only for seniors. There are companies who are interviewing for summer positions as well as full-time, year-round positions. Also, many companies will provide information about summer employment even if they are not actually interviewing for summer. This is a great way to get experience in the corporate world. Come into our office and check the recruiting binders for information. (14)

All seniors and graduate students seeking employment for next year are now invited to bring their one-page resumes to the Career Services Office (201 Nassau St.) to be included in resume books we will offer employers. The service is free to participating students, but you must complete a short registration form and clip it to the resume you submit. Further information about this service is available in Career Services at the Recruiting Desk. Bring your resume in ASAP, preferably by Mon., Oct. 24. Resumes submitted after that date can be included buy may not receive maximum exposure. All resumes must be limited to one page. The Career Services Office is open 8:45 a.m. to 4:45 p.m. (13)

A representative from the Darden Graduate School of Business, University of Virginia will be holding Information Sessions at Career Service on Mon., Oct. 31 from 10 a.m. to 1 p.m. Sign-ups at Career Services are required. (The sessions will be held in one hour intervals.) (14)

REGISTRAR

Deadline for grading option changes is Oct. 14. Completed forms must be submitted to the Office of the Registrar by 5 p.m.

CLASSIFIED ADS

STUDENTS interested in working as Tiger Patrols with the Department of Public Safety should contact Josh Hardt at x88464 or Public Safety at x83133.

HELP STOP WORLD HUNGER – JOIN THE CROP WALK! Join the walk to stop world hunger on Sunday, October 16th. For details and registration forms come to the SVC office, 22 Murray-Dodge.

OFFICE ASSISTANT – Part-time position for reliable, friendly person with good communication skills at Optometrists' office in the Princeton MarketFair. Afternoon, evening and weekend hours' perfect for college student. Call Pam at 520-1008.

CPR Classes available: Sat., Oct. 15, 9-4 p.m. (1/2 hour break for lunch). Call Health Education Office-information/registration.

FUNDRAISING!!! Choose from 3 different fundraisers lasting either 3 or 7 days. No investment. Earn $$$ for you group plus personal cash bonuses for yourself. Call 1-800-932-0528, ext. 65.

*****SPRING BREAK 95*****
America's #1 Spring Break Company! Cancun, Bahamas, Daytona &Panama! 110% Lowest Price Guarantee! Organize 15 friends and TRAVEL FREE! Earn highest commissions! (800) 32-TRAVEL.

FALL REACH OUT DAY! Enjoy a couple hours outdoors raking leaves for some elderly people in town who have asked for student help. sign-up alone (we'll match you up) or with friends. Go at 1 p.m. and rake a few hours. Sign-up in the SVC office, 22 Murray-Dodge.

NO GIMMICKS – EXTRA INCOME NOW!! Envelope stuffing – $600 - $800 every week. Free Details: SASE to International Inc.,1375 Coney Island Avenue, Brooklyn NY 11230.

DAILY CROSSWORD PUZZLE

Edited by Trude Michel Jaffe

ACROSS
1 Part of rye bread
6 Brass instrument
10 Pluck
14 Street show
15 Dill, in the Bible
16 Folk follower
17 — costs: by any means

4 Lyric
5 Capital of Northern Ireland
6 Loafer ornament
7 Monad
8 Stupefied
9 Dined
10 Swordsman
11 — numerals
12 Peace goddess
13 Pavarotti

42 Weight allowance
43 Sang joyously
45 The roadrunner's pursuer
46 Mocassin
48 Peruvian
49 Nary a soul

50 Shakespeare's Athenian
52 Pillar
55 Lubricates
56 Shadow
57 Italian noble family
58 Property transfer
60 Be in debt

Erik Jorgensen — *Princetonian*

Nash's most notable work deals with game theory, which attempts to predict how individuals will make decisions under uncertainty.

(Continued from page one)

generally look out for their own interests."

He offered a simple example in the interaction of cars and pedestrians. Either drivers must wait for pedestrians to cross the street or pedestrians must let cars have the right of way. The "game" is "finite" because only two solutions are available.

"What one (player) wants to do depends on what the other will do," Vanzandt noted.

The "game" reaches equilibrium when each "player," knowing what the other has decided, can make the best decision. If, for example, all pedestrians know they must wait for cars to pass, the drivers can choose to continue without stopping for those on foot.

Each side can make the optimal decision by anticipating what the other will do. Pedestrians, not expecting cars to stop, will not choose to cause accidents, while drivers, expecting to proceed quickly, will not choose to slow down.

"When (their) expections and (their) choices are consistent," neither side will want to deviate from the decision that creates this equilibrium, Vanzandt said.

Nash Equilibrium is the "basic conceptual framework for analysis of strategic interaction," economics professor Avinash Dixit explained. In competitive markets, individual businesses must consider their competitors' actions and reactions before making decisions.

The need for "strategic competition," Dixit noted, leads to "circular thinking." Nash equilibrium provides a way to "cut through the circle," he said. Each competitor's strategic choice is that which is best

(Continued on page nine)

NY Times Oct 12, 1994

3 Economists Share a Prize foı

✗50

John F. Nash

John F. Nash, by most accounts the rock on which the mathematics of game theory was built, was born in Bluefield, W. Va., in 1928. He went to college at the Carnegie Institute of Technology, now Carnegie-Mellon University, in Pittsburgh, switching from chemical engineering to mathematics after his freshman year in 1945.

His ascent into the academic elite was rapid. After receiving both a bachelor's and master's degree from Carnegie in 1948, he completed a Ph.D. in mathematics at Princeton University in just two more years. His Ph.D. thesis, "Noncooperative Games," which was published in the journal Annals of Mathematics, laid out the framework for much of modern game theory.

Mr. Nash introduced the distinction between cooperative games, in which binding agreements can be made, and noncooperative games, where binding agreements are not feasible.

He developed an equilibrium concept for noncooperative games that came to be known as the Nash equilibrium.'

Mr. Nash went to the Massachusetts Institute of Technology as an instructor in 1951, later being promoted to associate professor. Struck down by mental illness in the late 1950's, he resigned from M.I.T. and since then has spent most of his years at Princeton. For a time he was a visiting scholar at the Institute for Advanced Study there.

William E. Sauro/The New York Times

For most of the last quarter-century, he has been associated with Princeton as a "visiting research collaborator" without formal obligations to the institution.

He is said to lead a quiet life in the Princeton community, nurtured by friends and associates on the faculty. In deference to his wish for privacy, no details of his family life were made available.

John C. Harsaı

Born May 29, 192
John C. Harsanyi has
his entire career at t
of Business at the Un
fornia at Berkeley. I
ed a visiting profess
and a professor in 19
A leading scholar
ethics and social cl
sanyi is Flood Resea
Business Administra
and professor of eco
tus, at Berkeley.
"In more than f
scholarship, John
probed the idea of r
man affairs," wrot
Miles, the business sc
Dr. Harsanyi retired
"His work centers
puzzles. The first is t
of what it means for
to take ethical posi
moral judgments, an
of rational but disti
can properly choose
ternatives it faces," I
"The second has
ory," he continued,
the rigorous formul
priate behavior for
who are in conflict w
al persons. Profes
contributions to bot
been absolutely fund
Dr. Harsanyi's wo
social choice showe
culture adopts utilit
notion that one pers
can be measured ag
— then it has an ar
tools to use in choosi
native policies, laws

Lost Years of a Nobel Laureate

By SYLVIA NASAR

PRINCETON, N.J.

SEVERAL weeks before the 1994 Nobel prize in economics was announced on Oct. 11, two mathematicians — Harold W. Kuhn and John Forbes Nash Jr. — visited their old teacher, Albert W. Tucker, now almost 90 and bedridden, at Meadow Lakes, a nursing home near here.

Mr. Nash hadn't spoken with his mentor in several years. Their hour-long conversation, from which Mr. Kuhn excused himself, concerned number theory. When Mr. Nash stepped out of the room, Mr. Kuhn returned to tell Mr. Tucker a stunning secret: Unbeknownst to Mr. Nash, the Royal Swedish Academy intended to grant Mr. Nash a Nobel Prize for work he had done as the old man's student in 1949, work that turned out to have revolutionary implications for economics.

The award was a miracle. It wasn't just that Mr. Nash, one of the mathematical geniuses of the postwar era, was finally getting the recognition he deserved. Nor that he was being honored for a slender 27-page Ph.D. thesis written almost half a century ago at the tender age of 21.

The real miracle was that the 66-year-old Mr. Nash — tall, gray, with sad eyes and the soft, raspy voice of someone who doesn't talk much — was alive and well enough to receive the prize. For John Nash was stricken with paranoid schizophrenia more than three decades earlier.

Mr. Nash's terrible illness was an open secret among mathematicians and economists. No sooner had Fortune magazine singled him out in July 1958 as America's brilliant young star of the "new mathematics" than the disease had devastated Mr. Nash's personal and professional life. He hadn't published a scientific paper since 1958. He hadn't held an academic post since 1959. Many people had heard, incorrectly, that he had had a lobotomy. Others, mainly those outside Princeton, simply assumed that he was dead.

He didn't die, but his life, once so full of brightness and promise, became hellish. There were repeated commitments to psychiatric hospitals. Failed treatments. Fearful delusions. A period of wandering around Europe. Stretches in Roanoke, Va., where Mr. Nash's mother and sister lived. Finally, a return to Princeton, where he had once been the rising star. There he became the Phantom of Fine Hall, a mute figure who scribbled strange equations on blackboards in the mathematics building and searched anxiously for secret messages in numbers.

Then, roughly 10 years ago, the awful fires that fed the delusions and distorted his thinking began to die down. It happened very gradually. But, by his mid-50's, Mr. Nash began to come out of his isolation. He started to talk to other mathematicians again. He began to work on mathematical problems that made sense. He made friends with several graduate students. He didn't get a job, but he started to learn new things, like using computers for his research.

And here he was at Meadow Lakes. Within a few weeks, Mr. Nash got the early morning telephone call from Stockholm — 45 minutes late, as it turned out — telling him that he was being honored along with two other pioneers of game theory, John C. Harsanyi of the University of California at Berkeley and Reinhard Selten of the University of Bonn.

Alicia Nash, with whom Mr. Nash shares a home near Princeton even though the couple were divorced years ago and who was let in on the secret along with Mr. Tucker, breathed a sigh of relief. They called their son, also a mathematician, and Mr. Nash's sister to tell them the great news. Later, there were champagne corks and a news conference, dry Nashian jokes about the prize money not being all that good (his share is about $310,000) and consultations with other Princeton laureates about the proper way to address Sweden's King and Queen when the award is presented Dec. 10. There was even an invitation to visit the White House, on Nov. 28.

On one level, John Nash's story is the tragedy of any person with schizophrenia. Incurable, incapacitating and extremely difficult to treat, schizophrenia plays terrifying tricks on its victims. Many people with the disease can no longer sort and interpret sensations or reason or feel the full range of emotions. Instead, they suffer from delusions and hear voices.

But in Mr. Nash's case, the tragedy has the added dimensions of his early genius — and of the network of family and friends who valued that genius, wrapping themselves protectively around Mr. Nash and providing him with a safe haven while he was ill. There were the former colleagues who tried to get him work. The sister who made heartbreaking choices about his treatment. The loyal wife who stood by him when she no longer was his wife. The economist who argued to the Nobel committee that mental illness shouldn't be a bar to the prize. Princeton itself.

Together they made sure that Mr. Nash did not wind up, as so many victims of schizophrenia do, a patient in a state hospital, a homeless nomad or a suicide.

Schizophrenia usually strikes people in their teens or early 20's, often without warning, just as they are about to spread their wings. Mr. Nash was struck when he had already begun to soar.

Schizophrenia is often confused with manic depressive illness, the disease that afflicted Vincent Van Gogh, Virginia Woolf and a host of other geniuses. But that illness, primarily a disorder of mood rather than of thinking, typically arrives later in life. Sufferers can often hold high-level jobs and do extremely creative work between bouts. Schizophrenia, on the other hand, is too debilitating to co-exist with great accomplishment. Nijinsky, the Russian dancer, is one of the few known victims of schizophrenia other than Mr. Nash to have made his mark as a genius before the disease struck.

"It is always sad, but particularly when it involves someone as bright as he is," said John C. Moore, a retired mathematician who was close to the Nashes for 30 years. "These would have probably been his most productive years."

Mr. Nash has never talked about his illness publicly except to refer obliquely, at the news conference announcing his Nobel, to the fact that he had made some irrational choices in the past. He declined to be interviewed for this story, saying, "People know what they know."

But many of the people who have been close to him over the years or got to know him in the last few years have been willing, now that he has the extra protection of the mantle of a Nobel prize, to talk about his life and his achievements.

Starting Out

The First Signs Of Genius

John Nash's West Virginia roots are often invoked by people who knew him at Princeton or at the Massachusetts Institute of Technology, where he taught for a while in the 50's, to explain his lack of worldliness. But Bluefield, the town where he grew up, was hardly a backwater. It had the highest per capita income in the state during the 30's and 40's and was home to a handful of millionaires, the Virginia Southern railroad and a four-year Baptist college.

Mr. Nash's mother, Margaret, was a Latin teacher. His father, John Sr., was a gentlemanly electrical engineer. By the time John Jr. and his younger sister were in elementary school, in the middle of the Depression, the Nashes lived in a white frame house, down the street from the country club.

Nothing was more important to the senior Nashes than supervising their children's education, recalls the sister, Martha Nash Legg. John Jr. was a prodigy but not a straight-A student. He read constantly. He played chess. He whistled entire Bach melodies. He invented things and conducted experiments.

"John was always looking for a different way to do things," said

Mrs. Legg, a tall, handsome woman who is a potter in Roanoke. In elementary school, one of his teachers told John's mother that her son was having trouble in math. "He could see ways to solve problems that were different from his teacher's," Mrs. Legg said, laughing.

In the fall of 1945, Mr. Nash enrolled at Carnegie-Mellon, then Carnegie Tech, in Pittsburgh. It was there that the label "genius" was first applied to Mr. Nash. His mathematics professor called him "a young Gauss" in class one day, referring to the great German mathematician. Mr. Nash switched from chemistry to math in his freshman year. Two years later he had a B.S. and was studying for an M.S.

His graduate professor, R. J. Duffin, recalls Mr. Nash as a tall, slightly awkward student who came to him one day and described a problem he thought he had solved. Professor Duffin realized with some astonishment that Mr. Nash, without knowing it, had independently proved Brouwer's famed theorem. The professor's letter of recommendation for Mr. Nash had just one line: "This man is a genius."

Making Waves

Game Theory And More

In 1948, the year Mr. Nash entered the doctoral program at Princeton with a fellowship, the town was arguably the center of the mathematical and scientific universe. It not only had the Institute for Advanced Study and Albert Einstein, but also there was John von Neumann, the charismatic mathematician who helped develop the modern computer as well as the mathematical theory behind the H-bomb.

At once eager to prove himself and somewhat gauche, especially compared with older students who had served in the war, Mr. Nash quickly became one of the brilliant young men who performed mental pyrotechnics in the common room of Fine Hall. Soon after he arrived he invented an extremely clever game that was played with markers on hexagonal bathroom tiles. An instant fad in the common room, it was called "Nash" or "John." Parker Brothers brought out a version a few years later called Hex.

Other students found him a loner, odd as well as brilliant. When he wasn't in the common room talking a blue streak, he paced. Around and around he would go, following Fine Hall's quadrangular hallways, occasionally dashing into empty classrooms to scribble, with lightning speed, on blackboards.

"He was always an unusual person," said Jack Milnor, an undergraduate at the time and now a mathematician at the State University of New York at Stony Brook. "He tended to say whatever came into his mind."

Lloyd S. Shapley, then a graduate student and now a mathematician at the University of California at Los Angeles, added, "He was obnoxious. What redeemed him was a keen, beautiful, logical mind."

Mr. Nash's Nobel-winning thesis on game theory was the product of his second year at Princeton. Game theory was the invention of von Neumann and a Princeton economist named Oskar Morgenstern. Their 1944 book, "The Theory of Games and Economic Behavior," was the first attempt to derive logical and mathematical rules about rivalries. The Cold War and the nuclear arms race meant that game theory was an idea whose time had arrived.

Characteristically, Mr. Nash picked a problem for his thesis that had eluded von Neumann. Briefly, von Neumann only had a good theory for pure rivalries in which one side's gain was the other's loss. Mr. Nash focused on rivalries in which mutual gain was also possible. He showed that there were stable solutions — no player could do better given what the others were doing — for such rivalries under a wide variety of circumstances. In doing so, he turned game theory, a beguiling idea, into a powerful tool that economists could use to analyze everything from business competition to trade negotiations. "It wasn't until Nash that game theory came alive for economists," said Robert Solow, a Nobel laureate in economics at M.I.T.

Mr. Nash got his doctorate on his 22d birthday, June 13, 1950. After brief interludes as an instructor at Princeton and as a consultant at the Rand Corporation, the Cold War think tank, Mr. Nash moved on to teach at M.I.T. in 1951.

He arrived itching to show that he could solve really big problems. According to one story circulating at the time, Mr. Nash was in the common room knocking, as he often did, other mathematicians' work. An older professor is said to have challenged him to solve one of the field's most notorious problems.

The problem grew out of work done by G. F. B. Riemann, a 19th century mathematician, and was considered virtually insoluble. But Mr. Nash wound up solving it. To do so, he invented a completely new method for approaching the problem that turned out to unlock a difficulty encountered in a far larger class of problems. Mathematicians still describe the solution as "astonishing" and "dazzling."

Most mathematicians consider this and other work Mr. Nash did in pure mathematics to be his greatest achievements, worthy of Nobels if such were given in the mathematical field. Many joke that he got his Nobel for his most trivial work.

Gian-Carlo Rota, a mathematician at M.I.T. who is writing a chapter on Mr. Nash in his autobiography, said that Mr. Nash's results were so novel that they initially struck many people as incredible. "I heard Nash present his results on several occasions," said Professor Rota. "Each time, somebody in the audience would say, 'I simply don't believe a word of it.' "

The Disease

'It's All Over For Him'

By the mid-1950's, Mr. Nash was phenomenally productive. When he got tired of mathematicians, he would wander over to the economics department to talk to Mr. Solow and another Nobel laureate, Paul Samuelson.

And it was during this period that Mr. Nash met his future wife, Alicia Larde, an El Salvadoran physics student at M.I.T. who took advanced calculus from him. Small, graceful, with extraordinary dark eyes, Alicia looked like an Odile in "Swan Lake." "Very, very beautiful," recalls Ziporrah Levinson, the widow of Mr. Nash's mentor at M.I.T., Norman Levinson.

"He was very, very good looking, very intelligent," Mrs. Nash recalls. "It was a little bit of a hero worship thing." They were married in 1957, a year Mr. Nash spent on leave at the Institute for Advanced Study.

By the time the Nashes returned to M.I.T., John Nash had been awarded tenure. Mrs. Nash went back to graduate school and worked part time in the computer center. In the fall of 1958, she became pregnant with their son, John Charles Martin Nash. "It was a very nice time of my life," she recalled.

It is just then, when life seemed so very sweet, that John Nash got sick. Within months, at age 30 in the spring of 1959, Mr. Nash was committed to McLean Hospital, a psychiatric institution in Belmont, Mass., connected with Harvard University.

"Robert Lowell, the manic depressive poet, was also in the hospital," said Isador M. Singer, who shared an office with Mr. Nash at M.I.T., where he is now a professor. "There was Mrs. Nash, sitting there, pregnant as hell. Robert Lowell was sounding forth. And there was Nash, very quiet and almost not moving."

He added, "I've had that picture in my mind for years. I focused mostly on his wife and the coming child. I remember thinking, 'It's all over for him.' "

Psychiatrists who treat victims of schizophrenia ask people who haven't had the disease to imagine how they would feel if unseen voices shouted, if they lost capacity to feel or to think logically. And what if on top of that, asks E. Fuller Torrey, a psychiatrist in Washington and authority on schizophrenia, those closest to them began to avoid or ignore them, to pretend that they didn't notice what they did, to be embarrassed by their behavior? And what if the treatment was ineffectual? That is what happened to Mr. Nash.

In the months leading up to his hospitalization, Mr. Nash became another person. He skipped from subject to subject. Some of his lectures no longer made sense. He fled to Roanoke at one point, abandoning his classes. He wrote strange letters to various public figures.

"It was very sad," said Professor Shapley at U.C.L.A., who ran into Mr. Nash from time to time. "There was no way to talk to him or even follow what he was saying."

The months at McLean did little to arrest the disease. "Schizophrenia is a brain disease," said Dr. Torrey, adding that it is "a real scientific and biological entity as clearly as diabetes, multiple sclerosis and cancer are." But neuroleptics, the drugs that were used to treat some, but far from all, of the symptoms for the next several decades, were just coming on the scene. And psychoanalysis, which has since been discredited as a means of treating schizophrenia, was in vogue. The causes of the disease are still not known.

As absurd as it now seems, Mr. Nash's psychiatrists thought that Mrs. Nash's pregnancy was part of the problem and hoped that he would improve after the baby's birth. "It was the height of the Freudian period — all these things were explained by fetus envy," said Mrs. Levinson. Martha Legg added, sadly, "In those days, it was all supposed to be the mother's fault."

In any event, Mr. Nash's paranoia intensified and he could no longer work. After resigning his M.I.T. post, he went to Europe, wandering from city to city. He feared he was being spied on and hunted down and he tried to give up his United States citizenship. His wife and colleagues began to receive postcards with odd messages, many concerning numbers. "I rode on bus No. 77 today and it reminded me of you," one read. Eventually, the Nashes separated and he moved to Roanoke to live with his mother.

The Abyss

Two Decades Of Darkness

For most of the next 20 years, Mr. Nash divided his time between hospitals, Roanoke and, increasingly, Princeton.

In 1963, Mrs. Nash divorced him but eventually let him live at her house. Mr. Nash was hospitalized at least three more times. Mrs. Nash, who never remarried, supported her former husband and her son working as a computer programmer, with some financial help from family, friends and colleagues. "It was a pretty lean life," said Martha Legg.

Mr. Nash became a sad, ghostly presence around Princeton and a mysterious character, the Phantom of Fine Hall, in a novel set in Princeton's mathematics community, "The Mind-Body Problem" by Rebecca Goldstein (Penguin).

"Everyone at Princeton knew him by sight," recalls Daniel R. Feenberg, a Princeton graduate student in the 1970's and now an economist at the National Bureau of Economic Research. "His clothes didn't quite match. He looked vacant. He was mostly silent. He was around a lot in the library reading books or walking between buildings."

Alicia Nash believed very firmly, according to several people close to her, that Mr. Nash should live at home and stay within Princeton's mathematics community even when he was not functioning well. Martha Legg applauds her decision. "Being in Princeton was good for him," said Mrs. Legg. "In a place like Princeton, if you act strange, you're special. In Roanoke, if you act strange, you're just different. They didn't know who he was here."

Roger Lewin, a psychiatrist in Baltimore, agrees. "Some people are so disturbed that there is no way to get in touch with them, but for a significant group, compassion and receptivity of the surrounding community make all the difference."

Some former colleagues at Princeton and M.I.T. tried to help with jobs on research projects, though very often Mr. Nash couldn't accept the help. Professor Shapley at U.C.L.A. succeeded in getting a cash mathematics prize for Mr. Nash in the 70's. There were other forms of kindness, like getting Mr. Nash access to university computers or remembering to invite him to seminars when old friends turned up on campus.

Coming Back

Finally, A Remission

Still, the people who stayed in regular contact with him eventually came to believe that his illness would never end.

Then came what Professor Kuhn calls "a miraculous remission." And as happens, for reasons unknown, in the case of some people with schizophrenia, it was not, according to Mrs. Nash or Mrs. Legg, due to any drug or treatment.

"It's just a question of living a quiet life," said Mrs. Nash.

The most dramatic sign of that remission, perhaps, is that Mr. Nash was able to do mathematics again.

And now Mr. Nash is a Nobel laureate. The story of his prize is itself testament not only to his survival but to the fierce loyalty and admiration he inspired in others. During the 20-plus years of Mr. Nash's illness, game theory flourished and it is hard to find an important article in the field that doesn't refer to his work. But mathematicians and economists who were close to the secret deliberations say that the Nobel was hardly a sure thing.

By mid-1985, the prize committee was evidently actively considering an award for game theory. (The Nobel Memorial Prize in Economic Science is not one of the prizes established in the will of Alfred Nobel, but was created as a memorial to him in 1968.)

Five years later, the committee was making discreet inquiries not just about Mr. Nash's contribution but about his state of mind. There is no formal rule that a recipient must travel to Stockholm to accept the prize in person, give a Nobel lecture there or deliver a few profundities and words of gratitude to the King at the banquet. And there is certainly no rule that the recipient must hold a university post or have maintained an active career beyond the prize-winning contribution.

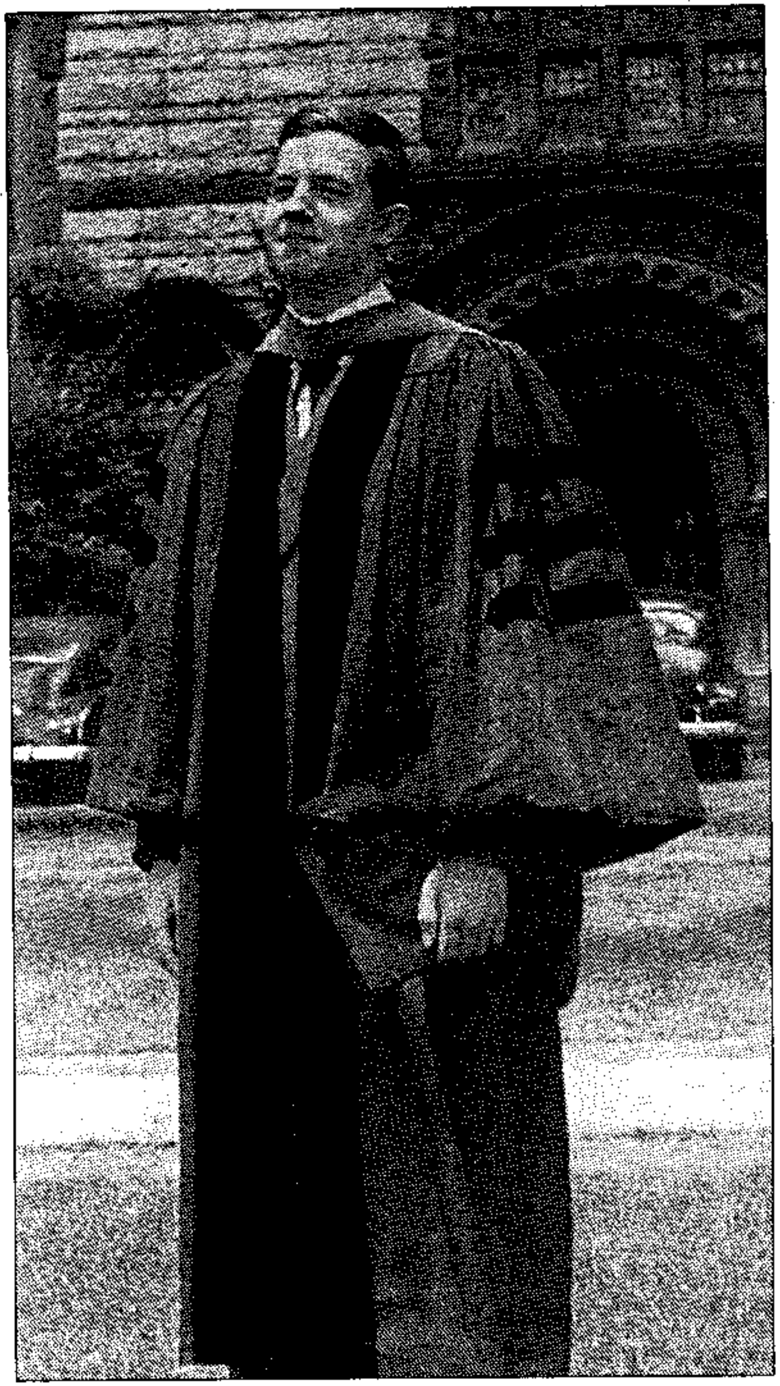
But no one wins prizes without an active constituency in his field. To most young game theorists who urged that he get a prize, Mr. Nash was a demigod. But Professor Kuhn played a particular role. A noted game theorist himself, he made it clear to the committee that it would be a grave injustice if Mr. Nash's illness cost him the prize.

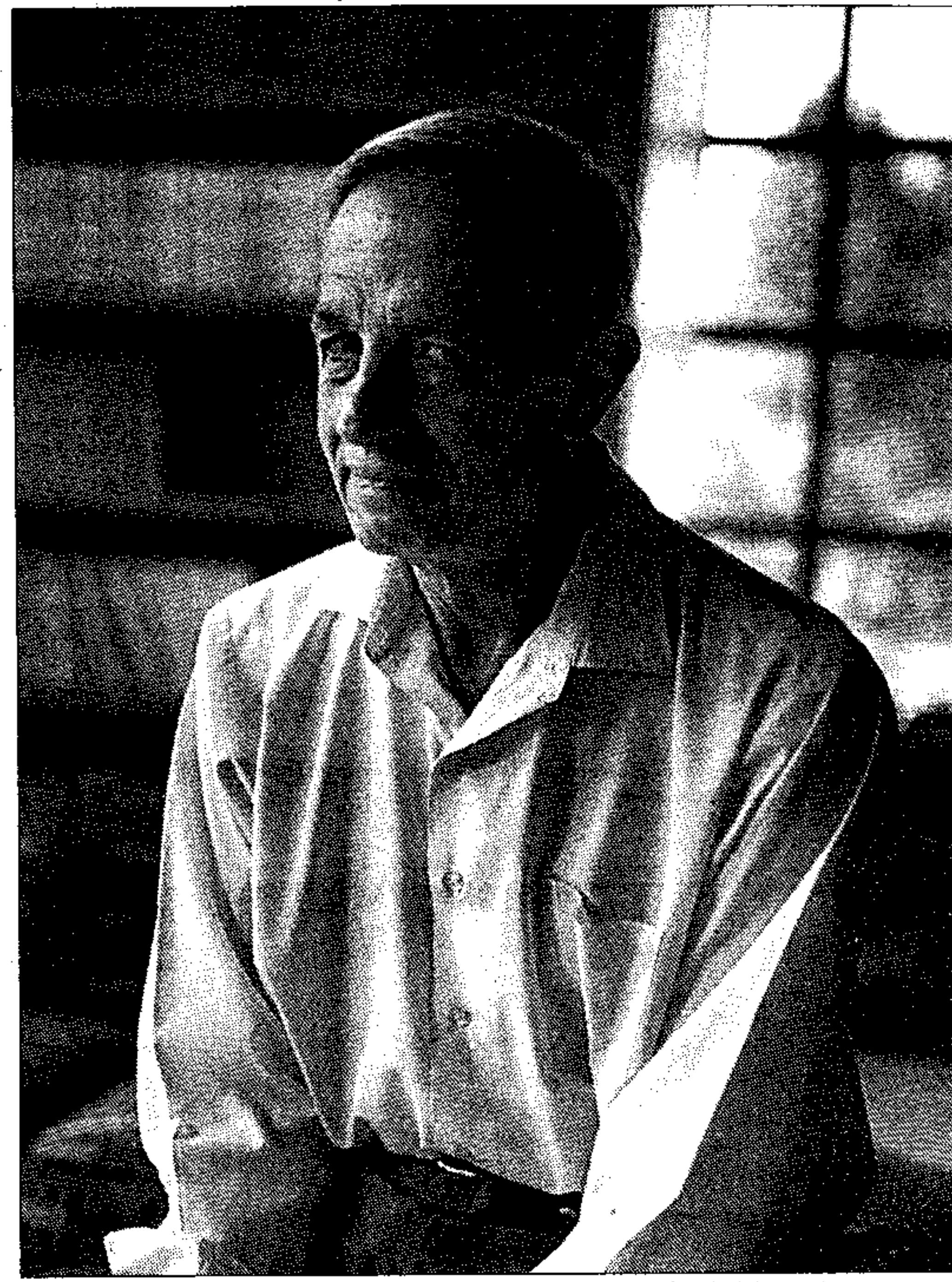
Early in September, Professor Kuhn got a clear signal that the prize would go to Mr. Nash when he was asked to prepare a curriculum vitae for him and to provide some photographs. At the professor's suggestion, Princeton created the title Visiting Research Collaborator to provide a ready answer for Mr. Nash to the inevitable question of his current affiliation.

The reaction to the announcement was jubilation. "The main message to the world is that the academy says mental illness is just like cancer, nothing special," said Ariel Rubinstein, a game theorist at Tel Aviv University. "It's great."

What will Mr. Nash do now? At 66, he is past the age when most mathematicians do their best work. But the researchers he now talks to say that he is interested in the major unsolved problems and that he has learned to use the computer in ingenious ways.

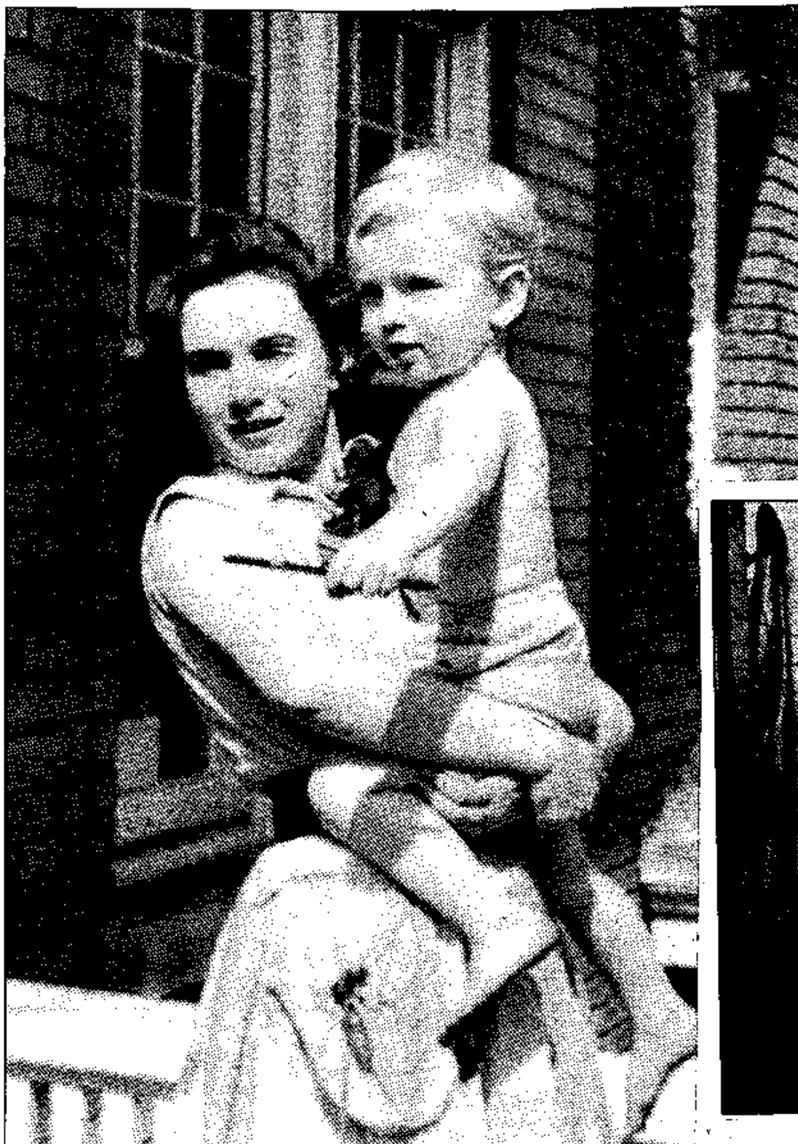
"The truths Nash discovered were all very surprising," said Simon Kochen, another Princeton mathematician. "Nash is a man who surprises people." ■





Courtesy of Martha Nash-Legg (left), Robert P. Matthews

John Forbes Nash Jr. in 1950, after receiving his doctorate at Princeton, and earlier this year, shortly before being named a Nobel winner.



Courtesy of Ma

Alicia Nash with her son around 1960.

Robert Mottar

John Nash in 1958, when Fortune called him a star.

Courtesy of Martha Nash Legg

John Nash with his sister and parents in the early 50's.

The New York Times

December 11, 1994 Sunday, Late Edition - Final

Letters

To the Editor:

What is a bit unusual about John Nash's experience was the level and quality of social support he received over the first 20 years of his schizophrenia. His family in Roanoke, Va., provided Mr. Nash with protection, privacy and refuge when he wanted it. More extraordinary was the devotion of his wife, Alicia, who, despite the break-up of the marriage, permitted Mr. Nash to live in her house in Princeton, N.J., while she supported their son and her ex-husband with some financial help from family, friends and colleagues.

But the most crucial factor that made Mr. Nash's survival and recovery possible was the place where he spent most of those 20 years. Mr. Nash had done his graduate work at **Princeton University**, and he and his wife returned to the Princeton community early in his illness. There, Mr. Nash was was revered by young graduate students because of the seminal work he had done and was not overlooked by his former colleagues who repeatedly offered him research work even though he was usually unable to accept their help. Mr. Nash was given unlimited access to libraries and computers and was invited to seminars when old friends turned up on campus.

It is impossible to overestimate the beneficial effect of the respectful and emotionally supportive atmosphere of the Princeton community on Mr. Nash's self-esteem, or the destructive effect that being discounted, disregarded and rendered all-but-invisible has on the morale of most similarly afflicted persons.

We do not, as yet, know what makes some patients have a remission of their disease while others do not. There is one thing, though, that the story of Mr. Nash tells us: the support of family, friends and others can help patients preserve their sense of self, faith in people and hope for the future, while waiting for their remission to happen.

IRWIN N. HASSENFELD
Albany, Nov. 19

The writer is professor of psychiatry at Albany Medical College.

GRAPHIC: Photo: John Nash lost two decades. (Reuters)

Letters

To the Editor:

Your article on John Nash ("The Lost Years of a Nobel Laureate, Nov. 13) was beautifully done. A five-minute conversation with Professor Nash leaves a person feeling smarter and with enough to think about for the next two weeks.

JAMES MANGANARO
Princeton, Nov. 18

The writer was one of Mr. Nash's students at M.I.T. in the late 1950's.

•

To the Editor:

You should be complimented on the article on John Nash. As a friend of Alicia Nash, I found the article sensitive, respectful and thoughtfully written. It will be treasured, I am sure, in the archives on John Nash, Nobel laureate.

ELIZABETH J. KEEGAN
Manhattan, Nov. 19

•

To the Editor:

What is a bit unusual about John Nash's experience was the level and quality of social support he received over the first 20 years of his schizophrenia. His family in Roanoke, Va., provided Mr. Nash with protection, privacy and refuge when he wanted it. More extraordinary was the devotion of his wife, Alicia, who, despite the break-up of the marriage, permitted Mr. Nash to live in her house in Princeton, N.J., while she supported their son and her ex-husband with some financial help from family, friends and colleagues.

But the most crucial factor that made Mr. Nash's survival and recovery possible was the place where he spent most of those 20 years. Mr. Nash had done his graduate work at Princeton University, and he and his wife returned to the Princeton community early in his illness. There, Mr. Nash was was revered by young graduate students because of the seminal work he had done and was not overlooked by his former colleagues who repeatedly offered him research work even though he was usually unable to accept their help. Mr. Nash was given unlimited access to libraries and computers and was invited to seminars when old friends turned up on campus.

It is impossible to overestimate the beneficial effect of the respectful and emotionally supportive atmosphere of the Princeton community on Mr. Nash's self-esteem, or the destructive effect that being discounted, disregarded and rendered all-but-invisible has on the morale of most similarly afflicted persons.

We do not, as yet, know what makes some patients have a remission of their disease while others do not. There is one thing, though, that the story of Mr. Nash tells us: the support of family, friends and others can help patients preserve their sense of self, faith in people and hope for the future, while waiting for their remission to happen.

IRWIN N. HASSENFELD
Albany, Nov. 19

The writer is professor of psychiatry at Albany Medical College.

•

To the Editor:

John Nash is not the first Nobel prize winner who "lost years" before his accomplishments were publicly recognized. Henri Dunant of Switzerland, who founded the International Red Cross in 1863 and was the primary force behind the Geneva Convention, was subsequently eased out of the organization, and the "founder" mantel was appropriated by Gustave Moynier. Dunant consequently went into an exile of poverty and isolation, where he was overcome with shame and plagued by nightmares of his misfortunes until the end of his life. When Dunant was finally vindicated and his name reinstated as the founder of the Red Cross, he could not appreciate winning the first Nobel Peace Prize in 1901, which he shared with Frédéric Passy, founder of the International League for Permanent Peace.

LOTTI S. TOBLER
Manhattan, Nov. 17

The writer is on the editorial board of The Swiss-American Review.

•

To the Editor:

Although the Nobel economics laureate John Nash almost never talked about his schizophrenia publicly and though he declined to be interviewed for your article, many of his friends, relatives and associates quite freely told you personal stories about him. Did any of these people check with Professor Nash to see whether he was willing to be have them discuss personal information he had chosen not to reveal? Or did they think that was beside the point?

FELECIA ACKERMAN
Providence, R.I., Nov. 14

The writer is a philosophy professor at Brown University.

•

To the Editor:

Your article perpetuates a dangerous equation of psychosis with schizophrenia, reflecting assumptions that contribute to erroneous diagnoses of schizophrenia in manic-depressive cases.

Psychosis is a state of brain dysfunction with increased dopamine brain activity that can occur both in schizophrenia and in a significant minority of manic-depressive individuals. Thus the impaired ability to think rationally, distorted perceptions and impaired ability to interpret sensations or the intent of others can strike both manic-depressive and schizophrenic patients.

The assumption that manic-depressive illness typically strikes later in life is not correct; many individuals experience the onset as adolescents, with its main symptom dysphoria rather than mania or depression — the person is irritable and difficult.

In addition, creativity occurs only when the manic-depressive person goes into remission or stabilizes at a mild low-grade manic level. This reflects a dopamine level that is too low to trigger psychosis. The schizophrenic person has a bizarre semblance of creativity when there is a psychotic loosening of associative thinking. Once the schizophrenic recovers from the psychosis, there is lack of original thinking as well as paucity of all thinking.

Manic depressive episodes follow psychosocial stresses (like a spouse's pregnancy) or physiological stresses (such as smoking marijuana, which lowers serotonin and triggers increased dopamine activity in the brain). All these factors are precipitants rather than causes of manic-depressive illness, which are genetic. The genetic causes consist of contributing to irregular synthesis of proteins like dopamine as well as receptor molecules.

LEO I. JACOBS
Lake Barrington, Ill., Nov. 14

The writer is a psychiatrist.

•

To the Editor:

Sylvia Nasar ridicules as "absurd" the hypothesis that Alicia Nash's pregnancy played a role in precipitating her husband's decompensation with a paranoid schizophrenic disorder in the 1950's. The idea of fetus envy is dismissed as Freudian nonsense.

Regardless of the degree to which the idea that a man may envy a woman's capacity to procreate is given much credence by psychoanalysts then or now, there is a widespread consensus among psychoanalysts and psychiatrists that having a first baby is often highly stressful for both husband and wife. There often are a wide variety of psychodynamic concerns, including feelings

Page One

John Nash
x50
-- MG
Pub: NY Times
Date: Dec 11, 1994

about moving to the status of a parent, capability of being a good parent, the possibility the baby will be abnormal and the degree of responsibility being shouldered.

The causes of schizophrenic disorders as well as their precipitating elements are unknown. Ms. Nasar has no basis for characterizing as absurd the theory that Mrs. Nash's pregnancy played a role in precipitating John Nash's decompensation.

In 1911, Freud described the Schreber case, in which a Dresden judge's appointment to a prominent position precipitated into a full-blown paranoid schizophrenic decompensation. For some individuals, life's becoming especially sweet and successful, paradoxically, constitutes an overwhelming stress.

JOSEPH S. SCHACHTER
Pittsburgh, Nov. 14

The writer is a psychoanalyst and psychiatrist.

•

To the Editor:

Sylvia Nasar's article is a great contribution to the understanding of schizophrenia. In disclosing the details of John Nash's tragedy, she provided a powerful, focused view of the emergence and impact of schizophrenia as it affects the victim, his family and others in his world.

Few articles have conveyed the human cost as effectively as did Ms. Nasar's. It will do more to eliminate the stigma of schizophrenia than sermons and pleas. She has allowed the facts to cry out — not merely speak for themselves.

Mr. Nash's affliction struck when science began to understand schizophrenia. Now, we are in a dynamic period of scientific progress where progress in neuroscience and molecular biology have developed new treatments and hope for cures.

The scientists who have been funded by National Alliance for Research on Schizophrenia and Depression the last seven years, are making unprecedented progress toward bringing normal and productive lives to those suffering from the severe mental illnesses. The goal of this work is that there be no more tragic loss of life's potential, such as that experienced by John Nash.

CONSTANCE E. LIEBER
Great Neck, L.I., Nov. 16

The writer is president of the National Alliance for Research on Schizophrenia and Depression.

•

To the Editor:

Your story was profoundly moving. "Sonnet to a Nobel Prize, Awarded After 30 Years of Schizophrenia" is my response:

Death's-head spiders drop along invisible
wires, out of walls, plugging holes
in motionless air unseen, unfelt, unknown,
filling poison bricks with metaphysical
venom, wrapping filmy tendrils tight
against a row of window panes, glass
sealed as hard as granite, doors plastered
shut, unlived-in rooms with blackened light.
Purple haze pouring over eyeballs.
Martian music winding through the rain.
Plastic hot balloons explode the brain,
frantic reptiles claw across the sky. All
bombs away: fires die: surviving
faces blink: incredible sunshine, warm, thriving.

BURTON RAFFEL
Lafayette, La., Nov. 13

The writer is the translator, writer and editor of many works.

X 50

News from
PRINCETON
UNIVERSITY

Communications and Publications
Stanhope Hall
Princeton, New Jersey 08544-5264
(609) 258-3600
(609) 258-1301 fax

Contact: Jacquelyn Savani
(609) 258-3601

For Immediate Release
Distributed Oct. 10, 1994

John Nash Shares 1994 Nobel Prize in Economics

Princeton N.J., October 10--John Nash, Visiting Research Collaborator at Princeton University, was today awarded the 1994 Nobel Prize in Economic Sciences for his seminal mathematical contributions to game theory. The prize was shared with John C. Harsanyi of Berkeley and Reinhard Selten of the University of Bonn, Germany. They won "for their pioneering analysis of equilibria in the theory of non-cooperative games."

Nash put forth his key idea--the Nash equilibrium--in the Ph.D. thesis he submitted to the Princeton Mathematics Department in 1950, when he was 22 years old. He had received his B.S. and M.S. degrees in three years, 1945 to 1948, at Carnegie-Mellon University in Pittsburgh (then Carnegie Institute of Technology). He did his Ph.D. work at Princeton in two years. The thesis was entitled "Non-cooperative Games."

In his Ph.D. thesis he defined a new concept of equilibrium and used methods from topology to prove the existence of an equilibrium point for n-person, finite, non-cooperative games. The "n" refers to the number of players; "finite" means the number of possible strategies are limited; and "non-cooperative" means no communication and therefore no collusion or side-payments are allowed between players.

So we have a game played by any number of players, unable to communicate or cooperate with each other. In the abstract formulation of game theory, a contest consists of each player choosing a pure strategy, which is a complete plan for every possible situtation that he or she might encounter during the course of play. When the pure strategies of all players are submitted to an umpire, the entire course of play and the payoffs to the players are determined. But all games cannot be solved with pure strategies; therefore players must use a mix of pure strategies by choosing the probabilities with which each pure strategy is played. Thus, in the game of "Matching Pennies," the pure strategies are "Heads" or "Tails," and the mixed strategies are the random frequences with which a player chooses to play these pure strategies.

Nash proved that there exists at least one set of mixed strategies, with one for each player--a Nash-equilibrium point--such that no player can improve his or her position by changing his or her strategy. At a Nash equilibrium point no one can improve his or her position, and therefore that profile of mixed strategies enjoys an essential property of stability. Although Nash proved the existence of such a profile of mixed strategies, it need not be unique.

(more)

Economists learned of game theory through the 1944 publication of *Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern. Their analysis was largely limited to games involving only two players. When analyzing games with more than two players, they assumed that players would form coalitions (for example, two ganging up against the third) thus reducing the number of players to two. This theory is called "cooperative" game theory. Nash, however, was able to deal with the general n-player game and to prove that points of equilibrium exist even if there is no cooperation or collusion between the players, which led game theory in a direction that von Neumann and Morgenstern had not imagined.

Nash's proof of the existence of at least one equilibrium point in this very wide class of non-cooperative games has had a major impact on modern economic theory. If we think of economic behavior as a game in which there are well-defined rules and all the players try to maximize their payoffs, then in general it will be possible for any given player to improve his or her position by changing his or her strategy. Consequently, players will keep changing their strategies until they reach a Nash equilibrium point at which no player can improve his or her position. This analysis makes it possible in some cases to predict the likely strategies that economic actors will adopt in the long run--namely, those at a Nash equilibrium point at which no player can change to improve his or her outcome.

A short version of Nash's Ph.D. thesis appeared as an announcement in *Proceedings* of the National Academy of Sciences in 1950; that version with its one-page proof was entitled "Equilibrium points in n-person games." The thesis, revised by Nash, was published in 1951 as "Non-cooperative Games" in *Annals of Mathematics*.

Initially, in the thesis, Nash used Brouwer's fixed point theorem (1926) to prove the existence of an equilibrium point. Both the *Proceedings* announcement and the *Annals* version use the more general Kakutani's Fixed Point Theorem (1946) in place of Brouwer. Nash credits this simplification of the proof to a suggestion by David Gale, now professor emeritus of mathematics at the University of California, Berkeley, then a graduate student in mathematics at Princeton.

Nash's thesis was supervised by Princeton Mathematics Professor Albert Tucker, a topologist turned game theorist. The same year as Nash's thesis, Tucker created the influential paradox known as the "Prisoner's Dilemma."

Prisoner's Dilemma as Example of Nash Equilibrium

For an example of the Nash equilibrium, let us look at the Prisoner's Dilemma. It depicts two partners in crime confronted with the following choices: if one confesses and the other does not, the confessor goes free and the other goes to jail for a long time; if neither confesses, each goes to jail for a short time; if both confess, each goes to jail for an intermediate length of time. Each reasons that he is better off confessing because if the other confesses, he receives an intermediate sentence by confessing and a long sentence by not confessing; if the other does not confess, he goes free by confessing

(more)

and receives a short sentence by not confessing. Since each reasons this way, each confesses, and so each is given an intermediate sentence; whereas if each had not confessed, each would have received a short sentence. The strategy whereby both confess is the Nash equilibrium in the game because neither can improve his position by changing his strategy (to renege on confessing means jail for a long time).

Nash was born in Bluefield, W.V., in 1928. He attended college as a Westinghouse Scholar. Having begun college as a chemical engineering major, he switched after a year to mathematics.

He was appointed research assistant and instructor at Princeton in 1950-51 and worked as a consultant for the RAND Corp. during the summers of 1950 and 1952. In 1951 he was appointed Moore Instructor at the Massachusetts Institute of Technology and promoted to assistant professor in 1953 and to associate professor in 1957. After resigning his professorship at MIT in 1959, he served as a research associate in mathematics at MIT in 1966-67.

Nash, who has lived in the Princeton area since the mid 1960s, is a Visiting Research Collaborator in the Mathematics Department at Princeton University, where he makes use of computing and library facilities in a program of independent research.

He held Sloan and NSF fellowships in the late 1950s and was a visiting member at the Institute for Advanced Study in Princeton in 1956-57, 1961-62 and 1963-64. More recently, he was awarded the von Neumann Theory Prize from the Operations Research Society of America and was elected a fellow of the Econometric Society. The Duke Mathematics Journal is currently planning a volume in his honor, which is being edited by Harold Kuhn and Peter Sarnak of Princeton and Louis Nirenberg of New York University.

The last person affiliated with Princeton to win the Nobel Prize in Economics was the late Sir W. Arthur Lewis, a professor in the Economics Department when he won in 1979.

Five current Princeton physicists have won Nobel Prizes: in 1963 Eugene P. Wigner, Thomas D. Jones Professor of Mathematical Physics, Emeritus; in 1977 Philip W. Anderson, Joseph Henry Professor of Physics; in 1980 Val L. Fitch, James S. McDonnell Distinguished University Professor of Physics, Emeritus; in 1993 Joseph H. Taylor, James S. McDonnell Distinguished University Professor of Physics, and Russell A. Hulse, principal research physicist at the Princeton Plasma Physics Laboratory. Toni Morrison, Robert F. Goheen Professor in the Humanities, won the 1993 Nobel Prize in Literature.

***Note:** Especially good for comment on Nash are Princeton economists Avinash K. Dixit (609-258-4013) and Ariel Rubinstein (609-258-4033); David Kreps of Stanford; and Robert J. Leonard of the University of Quebec in Montreal (517-987-4114).*

Game Theory Captures a Nobel

By PETER PASSELL

The 1994 Nobel Memorial Prize in Economic Science, a $930,000 award to be divided among three pioneers in the field of game theory, celebrates achievements in building the foundations for analyzing interactions among businesses, nations and even biological species.

But just as important, the prize awarded to John F. Nash of Princeton University, John C. Harsanyi of the University of California at Berkeley and Reinhard Selten of the University of Bonn acknowledges a sea change in economics that has occurred in the last two decades.

Economics has been a discipline dominated by the concept of perfect competition — competition among so many participants that no single buyer or seller need worry about the responses of others.

And perfect competition has proved to be a powerful idea, one that predicted how free-market economies would evolve and gave policy makers a reliable compass for figuring how best to encourage growth as well as a fair division of the economic pie.

But in a world of hostile takeovers, trade wars and big government, classical economics is giving way to game theory, an approach that focuses on the give and take among "players." While classical economics works for the international market in wheat with thousands of buyers and sellers, it takes game theory to try to figure out how Safeway will change the price of English muffins if the A.& P. marks down bagels.

Game theory "opens up terrain for systematic thinking that was previously closed," said Paul Krugman of Stanford University, who has applied game theory to world trade.

John von Neumann and Oskar Morgenstern, economists at Princeton, invented the field. Their book published in 1944, "The Theory of Games and Economic Behavior," was the first to delve deeply into the likely consequences of strategic interactions, where all the actors must consider the potential for reaction. Both men are dead, and, therefore, not eligible for the economics prize, which is not one of the five awards established in the will of Alfred Nobel but rather a special prize created in 1968 as a memorial to Nobel.

John Nash, who received a Ph.D. from Princeton in 1950, is widely credited with laying out the formal mathematical principles of "games" — think of them as rivalries — in which everyone knows what everyone else knows and everyone is motivated by self-interest.

"Nash is the point of departure" for all modern game theory, argues Avinash Dixit, an economist at Princeton and a co-author of "Thinking Strategically," the first guidepost for predicting the consequences of rivalries.

One glaring limitation of Dr. Nash's work is the assumption about perfect knowledge of rivals' motives and resources. Compaq does not know exactly what Apple is prepared to invest to build a better laptop computer. For that matter it does not even know whether other companies are preparing to jump into the market, and under what circumstances. And here the work of John Harsanyi, a Hungarian-born mathematical economist, filled the theoretical breach in the late 1960's.

"Harsanyi gave shape to the fog in real-world games," said Barry Nalebuff of the School of Organization and Management at Yale. In the Harsanyi world, nothing need be known for certain as long as it is predictable in terms of chance. Thus when Compaq and Apple are figuring pricing strategies, they need only assign probabilities to the other's uncertain responses and counterresponses.

Another limitation to the Nash approach is that it did not offer insight into what would happen if more than one ending to the game was possible, even if the players acted consistently and in their own best interests. It does not stretch the imagination, for example, to think of the outbreak of World War I as only one of many plausible consequences to the diplomatic and military maneuvering in August 1914.

The German economist Reinhard Selten enriched the Nash model in 1965 by offering theories for discriminating between game outcomes that are reasonable and unreasonable. The mathematics is quite complex, but some of the underlying ideas are intuitive. For example, an outcome dependent on someone's taking an unreasonable threat seriously (as in "buy my rug for $200 or I will kill your first-born child") may be discarded.

All this may seem as abstract and impractical as the theory of perfect competition — and for many years it was dismissed as just that. Nonetheless by the 1970's many economists were turning to game theory for inspiration, if only because they lacked answers to questions that turned on strategic behavior.

Thomas Schelling, an economist now at the University of Maryland who is in a class by himself in applied game theory, decades ago introduced ideas like the strategic value of brinkmanship. Indeed, some think he ranks with Dr. Nash as a founder of the field.

The great bulk of work by economists in game theory has been in an area where its insights had been most sorely missed: the organization of industry.

Robin Wells, an economist at the Stanford Business School, offers an example. Intel, the microprocessor giant, gave up an effective monopoly on the 86-series chip by allowing Advanced Micro Devices to share the technology. Intel, it seems, decided that computer makers would not lock themselves into a new microprocessor technology unless they were protected from future price-gouging by a monopolist. So by licensing another manufacturer, Intel successfully increased the demand for its own product.

Here, game theory explained corporate behavior that made no sense in nonstrategic terms. Game theorists have also been hired to create corporate strategy from scratch, most notably in the case of the Federal Communications Commission's auction of bands on the radio spectrum for use in wireless communications. "Every major bidder hired academic game theorists as consultants," said Andrew Schotter, an economist at New York University.

In the view of Mr. Schotter, however, the main value of game theory in formulating corporate strategy is more modest: getting executives to think carefully about response and counterresponse in the marketplace. The purer gold, Mr. Schotter believes, will be mined by "institution builders" who must make rules to induce cooperative behavior. Corporations, for example, need to create incentives to minimize the conflict between their own interests and those of their employees.

What works in the private sector might work in the public sector: The F.C.C. did, in fact, hire game theorists to set the rules for the spectrum auction. And the potential is far broader — changing the tax code to induce voluntary compliance, for example, or designing monetary policy to gain the most credibility for the Federal Reserve as an inflation fighter with the least risk of setting off a recession.

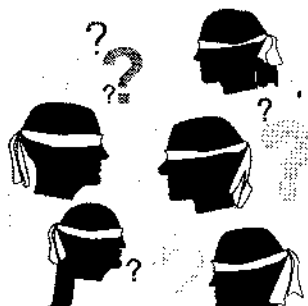
3 Economists Share a Prize for Insights Into How Rivalries Function

The Real World of Game Theory

Game theory will undergo a practical demonstration in December when the Federal Communications Commission begins to auction radio licenses for new wirelesss personal communications services.

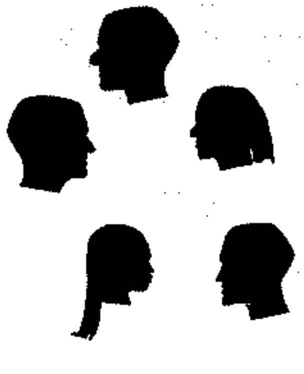
The F.C.C.'s goal is to raise the maximum amount of money, at least $10 billion, and it will use game theory to reach that target.

Game theory focuses on how "players" in economic "games" behave when, to reach their goals, they have to predict how their opponents will react to their moves. The concepts developed by the Nobel winners were used by other game theorists to devise the auction rules. — *EDMUND L. ANDREWS*



Coping With the Winner's Curse

Like poker, auctions are a 'game of incomplete information.' Bidders have to speculate about both the value of the good being sold and their opponents' strategies. The great fear is the 'winner's curse' of paying too much. But if all bidders are overly cautious as a result, the auctioneer runs the risk that the bids will be below the value of the good being sold.



Maximizing Information

Game theory says bidders become more confident with more information, so the F.C.C. has designed the auctions to be as open as possible. Unlike most auctions, where goods are sold one after the other, bidding for all the F.C.C. licenses will occur simultaneously. By seeing how other licenses are being valued, and how their opponents are bidding, bidders can react without fear of paying too much. In such a case, the F.C.C. hopes, the proceeds will meet its goal.

The New York Times

John F. Nash

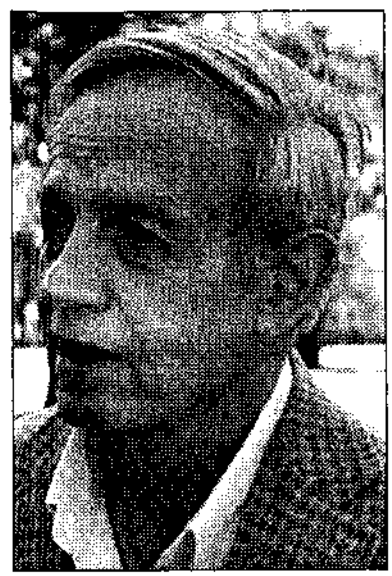
John F. Nash, by most accounts the rock on which the mathematics of game theory was built, was born in Bluefield, W. Va., in 1928. He went to college at the Carnegie Institute of Technology, now Carnegie-Mellon University, in Pittsburgh, switching from chemical engineering to mathematics after his freshman year in 1945.

His ascent into the academic elite was rapid. After receiving both a bachelor's and master's degree from Carnegie in 1948, he completed a Ph.D. in mathematics at Princeton University in just two more years. His Ph.D. thesis, "Noncooperative Games," which was published in the journal Annals of Mathematics, laid out the framework for much of modern game theory.

Mr. Nash introduced the distinction between cooperative games, in which binding agreements can be made, and noncooperative games, where binding agreements are not feasible.

He developed an equilibrium concept for noncooperative games that came to be known as the Nash equilibrium.'

Mr. Nash went to the Massachusetts Institute of Technology as an instructor in 1951, later being promoted to associate professor. Struck down by mental illness in the late 1950's, he resigned from M.I.T. and



William E. Sauro/The New York Times

since then has spent most of his years at Princeton. For a time he was a visiting scholar at the Institute for Advanced Study there.

For most of the last quarter-century, he has been associated with Princeton as a "visiting research collaborator" without formal obligations to the institution.

He is said to lead a quiet life in the Princeton community, nurtured by friends and associates on the faculty. In deference to his wish for privacy, no details of his family life were made available.

Economist Nash takes the Prize

Nobel committee cites West Windsor eccentric

By Laurie Lynn Strasser
Staff Writer

The world now knows John Nash as a Nobel Prize-winner, but the Dinky rail line crew knows him as "Sneakers."

Dr. Nash, who developed an economic predictive tool called "non-cooperative game theory" in his 1950 doctoral dissertation for Princeton University, was jointly awarded the Nobel in economics with two others who refined his work: John Harsanyi of the University of California at Berkeley and Reinhard Seiten of Bonn, Germany.

Dr. Nash has shuttled into Princeton Borough from his home on Alexander Road in Princeton Junction nearly every day for at least two decades.

When Dinky engineer Paul Connelly was shown a photograph of the new Nobel Laureate, he reacted instantly with a single word: "Sneakers." He said he had "never seen anybody smoke a cigarette as fast" as Dr. Nash, who silently reads the paper each day during his five-minute commute.

"He's known as Sneakers, simply because, no matter what the weather is, he's always wearing his sneakers," said conductor John Washburn. "For lack of a proper name, we just gave him a nickname. I've never seen him wear anything other than that, and I've been on and off this run for 23 years."

Each morning, those sneakers carry Dr. Nash from the Dinky Station to the Institute for Advanced Study in Princeton Township, where he spent three one-year stints as a visiting scholar between 1956 and 1963.

"He has had no formal association with us since then," said Norman McNatt, public relations officer for the Institute on Olden Lane. "It is often the case where people who are brilliant and creative go through a period where they produce monumental things, and then the rest of their lives are often dealing with the consequences of those great ideas in some way."

"Four days out of five," Dr. Nash comes to the institute in the morning, eats lunch and takes afternoon tea, but he is not on the payroll, Mr. McNatt said.

At Princeton University, Dr. Nash has the title of "visiting research collaborator," which means he can use the library and computers, said Justin Harmon, the university's director of communications.

Contacted at home Wednesday, Dr. Nash politely declined to give an interview, claiming he was refusing all such requests so he would not exclude anyone.

Born in 1928, he earned bachelor's and master's of science degrees in 1945-48 at Carnegie-Mellon University in Pittsburgh. He moved to the Princeton area in the 1960s after reportedly suffering a mental breakdown.

A recent Boston Globe article said he is "perhaps not entirely in his right mind." Wednesday's New York Times reported that he quit the faculty of the Massachusetts Institute of Technology after being "struck down by mental illness in the mid 1950s."

At a press conference Tuesday in Princeton's Jadwin Hall, an array of pens protruded from the Bluefield, W.V., native's shirt pocket.

On his feet was a trademark pair of ratty white canvas basketball shoes.

Dr. Nash said he was unsure whether his current work could accurately be termed "research" because it is not particularly goal-oriented. As he phrased it, he is working on "impossible problems." He said he has a tendency to work on his own.

Asked what he thought about his theory's widespread application today, Dr. Nash responded, "Well, I see my name cited more often."

Asked how he feels about winning the Nobel Prize, he said, "Well, the money could be better."

He acknowledged no other income than the $300,000 he will receive for winning the Nobel. He said he did not care to comment on his personal finances or how he planned to use the prize money, because it "might affect his credit rating."

Dr. Nash said he found all the media attention "flattering" and somewhat embarrassing.

Dr. Nash said Joseph Kohn, chairman of Princeton University's Mathematics Department, had warned him that the Nobel committee in Sweden had a tendency to break the news very early in the morning, so he was awake at 6 a.m. beside the phone.

"I should have stayed in bed until 7 because the call came at 7:45," he said.

Dr. Kohn, who studied under Dr. Nash as an undergraduate at MIT, praised his former professor at the press conference.

"Newton is supposed to have said he could see further because he stood on the shoulders of giants," he said. "A whole generation of mathematicians have seen further because they have stood on the shoulders of John Nash."

Payoff point set forth by Nash theory while at PU

All players know the rules of the game.

When they have changed tactics all they can to optimize their payoff, they have reached the Nash equilibrium point.

The man for whom the point is named is John Nash, who won the Nobel Prize for his doctoral work in game theory.

The above explanation of his theory is a simplified version, said Princeton University spokeswoman Jacquelyn Savani.

Dr. Nash did not invent game theory, but he expanded upon it significantly, she said. Before he came along, "cooperative game theory" involved just two opposing sides or alliances of players, Ms. Savani said.

Dr. Nash developed a way of dealing with an arbitrary number of opponents, she explained. He proved that even though "every man is out for himself," there still are equilibrium points, as long as the game has a finite number of players, she said.

Today, game theory is used by economists to forecast which long-term strategies will be adopted. Biologists also utilize it to understand how natural selection operates on populations, both within and among species, Ms. Savani said.

— Laurie Lynn Strasser

Page one

University's Nobel count grows to 28 since 1927

By Laurie Lynn Strasser
Staff Writer

If John Nash's Nobel prize is a feather in Princeton University's cap, then that cap is a veritable headdress.

Princeton University affiliates, including alumni, visiting professors and faculty, have won 28 Nobel Prizes since 1927, according to statistics released Wednesday by University Communications Director Justin Harmon.

"Of institutions nationally we have quite a good share of these things," he said. "It's because we have remarkable faculty and remarkable community members. We haven't done a comparison list with University of Chicago, MIT(Massachusetts Institute of Technology) or Harvard, but it's a significant chunk."

Eighteen of Princeton's Nobel winners won in physics, three in chemistry, three in literature, two in economics, one in medicine and one in peace.

The most famous winner was Woodrow Wilson, a member of Princeton's class of 1879 and university president from 1902-1920. He won the Nobel Peace Prize in 1919 while he was president of the United States.

Albert Einstein does not technically number among Princeton's Nobel Prize winners. Although his office was in Jones Hall on campus when he received the Nobel in 1921, he was working for the Institute for Advanced Study at the time.

Last year, the university won more Nobels in a single year than ever before. A Nobel in Literature went to Author Toni Morrison, a humanities professor, and Nobels in physics were claimed by Joseph Taylor, a professor, and Russell Hulse, a researcher at the Plasma Physics Laboratory.

The only other Princeton affiliate besides Dr. Nash to win the Nobel for economics was emeritus professor Sir Arthur Lewis in 1979.

John Bardeen, who received his Ph.D. from Princeton in 1936, is one of only three people ever to win two Nobel prizes. He earned his Nobels for physics in 1956 and 1972. The other two double-Nobel laureates were Marie Curie and Linus Pauling.

Nobel prize winners are selected by the Royal Swedish Academy of Sciences, based in Stockholm. The $300,000 in prize money is awarded by the Bank of Sweden in memory of Alfred Nobel.

Biography Published Of Nobel-Winner John Forbes Nash

A biography of John Forbes Nash Jr., a mathematical genius who went mad, recovered late in life, and won the Nobel Prize in Economics in 1994, has been written by New York Times economics correspondent Sylvia Nasar and published by Simon & Schuster.

"*A Beautiful Mind* is a splendid book, deeply interesting and extraordinarily moving, remarkable for its sympathetic insights into both genius and schizophrenia," said Oliver Sacks. "It is equally gripping as a portrait of the mathematical community at Princeton and of Nash's friends and family, and the perhaps crucial part they played in his psychic survival and eventual reemergence."

Born in 1928 in Bluefield, West Virginia, Dr. Nash earned his bachelor's and master's degrees in three years. He was sent off to Princeton's graduate program with the recommendation, "This man is a genius."

The handsome, arrogant, and highly eccentric 20-year-old Nash arrived in what was then "the center of the mathematical universe," where he mixed with such intellectual giants as Einstein, von Neumann, Morganstern, Artin, Church, and Lefschetz.

Dr. Nash avoided classes, read as little as possible, and spent most of his time in apparent idleness. But within 14 months he had solved a 100-year-old problem in economics, invented an ingenious topological board game later called Hex, and written the 26-page Ph.D. thesis, "Non-Cooperative Games," that won him a Nobel Prize nearly a half century later.

At age 30, just as he was about to be made a full professor at MIT, Dr. Nash experienced his first episode of paranoid schizophrenia, the most devastating and baffling of mental illnesses. For the next 30 years he suffered from severe delusions, hallucinations, disordered thoughts and feelings, and a broken will.

Struck Down at Zenith

He was at the height of his career, and had recently married a beautiful young physicist who never abandoned him. Although divorced, they live together today in Princeton Junction with their son, John, a brilliant mathematician who also suffers from schizophrenia.

Dr. Nash wandered the Princeton campus in the 1970s and 1980s, and caused riders on the Dinky to wonder at the identity of this oddly dressed man with the sad, immobile face. He had become known to University students as "The Phantom" because of his eccentric behavior just as his name, ironically, began to resurface in academic circles.

Sylvia Nasar writes, "Nash's insight into the dynamics of human rivalry — his theory of rational conflict and cooperation — was to become one of the most influential ideas of the twentieth century, transforming the young science of economics the way that

Continued on Next Page

BROTHERS IN ARMS: Jack Shakuro, a member of the U.S. Army's 82nd Airborne Division, spends a moment with his brother, Stas Hvoorikov, a Princeton firefighter, during the annual Princeton Fire Department parade last Saturday. The brothers are Russian emigres who now call Princeton home.

(Photo by Bill Allen/NJ SportAction)

For Grad School Alum File

The Man Behind a Beautiful Mind

The real John Nash never saw visions, and after 1970 he never took medication. But his love affair with Alicia, he says, is 'just like a movie.' BY SYLVIA NASAR

SINCE "A BEAUTIFUL MIND" opened, people who loved the movie but know that it is a fictionalized version of my 1998 biography of Nobel laureate John Nash have been asking me about the real Nash: How sick was he? How did he recover? How is he now?

A couple of weeks ago, Nash and I were at a cocktail party at the Institute for Advanced Study in Princeton. After regaling his listeners with box-office results, Nash joked that he hoped Universal was doing a better job of keeping its books than Enron. Watching this 73-year-old gentleman, you'd find it hard to believe that 15 years ago he was so sick that he was haunting the Princeton campus in mismatched plaids, speaking to no one, afraid to look anyone in the eye, his front teeth rotted.

In some ways, Nash's illness was a classic case of paranoid schizophrenia. Some of his peers were convinced that the early stages of the illness manifested themselves in graduate school, but the full-blown symptoms did not erupt until he was 30. In 1959, just as he was about to be promoted to full professor at MIT, he told the chairman of a rival department that he wouldn't be able to accept an offer because "I am scheduled to become the emperor of Antarctica." Convinced that he was "a messianic figure of great but secret importance," he frantically scanned The New York Times for encoded messages from aliens, and fiddled with radio dials to pick up signals from space.

JAN COLLSIÖÖ—PRESSENS BILD

LAUREATE: Nash accepts the Nobel Prize in Economics in Stockholm, 1994

At some point, he began hearing voices. Though he didn't literally see the figures who were shouting at him, the voices were as real to him as people on the street. As a young mathematician, he saw mathematical solutions—nonrational flashes of intuition—long before he could work out the reasoning. After the delusions and hallucinations took over, he said, "My ideas about supernatural beings came to me the same way my mathematical ideas did. So I took them seriously."

Like fewer than **one in 10** individuals, Nash 'emerged from irrational thinking,' as he puts it, **without medicine**

Nash responded dramatically to treatments available in the 1960s—crude and sometimes dangerous as these were. He was hospitalized half a dozen times, always involuntarily. At Trenton State, where he was incarcerated in 1961, he was injected with insulin and put into a coma daily for six weeks by physicians who hoped to shock Nash's brain back to health. Later he was treated with antipsychotics like Stelazine. He described the insulin shock as "torture," and blamed Stelazine for making him "foggy" (both true).

Like so many people who suffer from schizophrenia, Nash did not believe that he was sick. As his illness deepened, he accused his wife, Alicia, of wanting to lock him away. Exhausted and depressed, struggling to raise their son, Alicia obtained a divorce in 1963. In 1965 he moved to Boston, where he hoped to begin his life afresh. But he stopped taking medication, relapsed and finally wound up living in Roanoke, Va., with his mother. At 40, gray-haired and frail, he saw "a cadaver almost" when he looked at himself in the mirror. He spent his days sipping Formosa oolong in his mother's apartment.

Nash never stopped pining for Alicia, and she never really let him go. After his mother's death in 1970, he wrote to Alicia and begged her to shelter him. Astonishingly, she agreed. He moved back to Princeton, where students knew him only as the Phantom of Fine Hall, a mute figure who scribbled weird but witty messages on blackboards: MAO TSE TUNG'S BAR MITZVAH WAS 13 YEARS, 13 MONTHS AND 13 DAYS AFTER BREZHNEV'S CIRCUMCISION.

Moviegoers will be surprised to learn that powerful new drugs like clozapine played no role in Nash's recovery. Another kind of chemistry apparently did, however. Like fewer than one in 10 individuals who suffer from chronic schizophrenia, Nash "emerged from irrational thinking ultimately without medicine other than the natural hormonal changes of aging," as he later put it. No one knows why a lucky minority experience a dramatic lessening of symptoms in late middle age.

Even today Nash sometimes hears the old voices. But now he has learned to ignore them. "It's like a continuous process rather than waking up from a dream," he has said. Other Nobel laureates fly first class or start charities with their prize money. For Nash, who is doing research again, the most prized emoluments are simpler: being able to afford a cup of coffee at Starbucks, getting a driver's license and, most important, providing for his family—including a son who also suffers from schizophrenia—once more.

"A second take!" he quipped before kissing Alicia when they remarried last June. *"Just like a movie."*

NASAR *teaches journalism at Columbia University.*